How to tell mathematics –
Remembering Sofya Kovalevskaya

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I once made a list of a hundred and seventy infinitive verbs mode that were supposed to describe the work of a mathematician. Here are the beginning and the end of this list:

to attend to hear to discuss to think … to start again

(my translation)[1]. In this article, I will add the verb to tell (in the sense of telling a story, like the French *raconter* or the Italian *raccontare*, rather than the Frenchitalian *dire*)–a verb that was not yet in my list. And I shall address the question "How to tell mathematics?" Not the usual (but highly nontrivial) "How to write mathematics?" I am not thinking of explaining a technical result in a specialized seminar but indeed I want to tell mathematics as a story.

I know at least three good reasons to discuss this question:
1. Mathematics, as an intellectual human endeavor, is part of our culture.
2. Most mathematicians (including me) get (rather high) salaries coming mainly from the taxes all citizens pay. It seems reasonable for them to try and explain what they do to those citizens.
3. We (here I mean people) use more and more sophisticated tools, from nuclear stations to cell phones through washing machines. Scientific popularization is a democracy issue: how can we, e.g., vote to choose the energy policy if we don't know at all how it works?

To the "How to write mathematics" question, one of my advice is: find a good example to illustrate, both the problem you investigate and its solution, then explain it in detail[2]. In this (written) text, I shall follow this advice and concentrate mainly on my book *Souvenirs sur Sofia Kovalevskaya* (English translation *Remembering Sofya Kovalevskaya*) [1]. To be consistent, I shall also try to explain why this is a good example...

Why Sofya Kovalevskaya?

- She did interesting, exciting mathematics. By "exciting", I mean that they are still interesting, more than a hundred and twenty-five years later: they contain questions that are still open.
- She also wrote some literary texts: articles for newspapers, plays, a novel (*Nihilist Girl*) [13] and childhood memories (*A Russian childhood*) [12]. She herself lived a "romantic" life, actually she lived many different lives...
- The history of her scientific reputation is quite interesting and useful...

As a mathematician, I am a specialist of integrable systems. In the 90's, I had an idea to study the topology of some of these systems. To test it, I was looking for an intricate example. I chose the Kovalevskaya top. I worked on this with Robert Silhol, who knew the real algebraic techniques it required. We wrote and published a paper on that subject [7]. I was only interested in counting the "Liouville tori" and investigating their bifurcations, I did not care for the mathematician who invented this wonderful example. The way she found it was also quite

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1   The complete list can be found in [4].
2   See [2].

remarkable, as it raised a precise form of the question "what is integrability?" the answers to which are not yet completely understood.

This was my (exclusively mathematical) background on Sofya Kovalevskaya in 2005 when I met Jean-François Peyret, who was staging a play about her. And this is how I began to be interested in her other lives.

Why not?

Well... because there were already quite a few books about her. I shall not make a list but only mention two of them, both on the American side[3].
- The Ann Hibner Koblitz biography, *A convergence of lives*, a really impressive book [11],
- *The Mathematics of Sonya Kovalevskaya*, by Roger Cooke [9].

The trouble was that the biography did not emphasize much on mathematics and that the Cooke book, which appeared in 1984, did not emphasize much on integrable systems, because it was written a little too early: the renewed interest of working mathematicians for these systems began more or less at the same time.

Although I loved these two books, as they are very interesting and turned out to be very useful for me, I must add, in view of the context of the present article, that they are academic works written in the academic style.

Kovalevskaya, definitely.

Sofya's many lives. She was (at least) a mathematician, a nihilist, a woman, a writer (I used the alphabetic oder). I felt involved with her posthumous life as well: she was very well integrated in the European mathematical community of her time (Germany, France, Italy, Sweden...), but her scientific reputation today is not at the level of her achievements. This I really wanted to discuss: at this point, history is a way of improving things, right?

Sofya's life. As she was very famous in her time, some of the stories she told in *A Russian childhood* and (less fortunately) what Anne Charlotte Leffler wrote in her 1895 biography [14] became legendary, in a way. I am particularly thinking of the story of the Ostrogradski calculus book that was used as wallpaper in her bedroom. How could I resist beautiful formulas surrounding the little girl–better surrounding than the usual pale pink roses... And Beethoven's Pathetic sonata she played to charm Dostoyevsky? The way she invented the sine function, her "white marriage" and her subsequent travel to Germany, her visit to Paris during the 1871 Commune, the ugly hat she wore when she went to visit Karl Weierstrass for the first time, how he burnt her letters and papers when he heard she was dead...

And what I knew of her mathematics, the astonishment of Weierstrass when she found a counter-example to a statement he (and everybody else, including Cauchy) always thought was true, and when she proved the so-called Cauchy-Kovalevskaya theorem; her investigation of the rigid body which gave her both the Kovalevskaya top, a new integrable system and the Bordin prize she was awarded by the Paris Academy of sciences...

So, what would I do with that? *Racontare*, to tell. To tell of this. Including the mathematics. To treat the "legends" as stories, as tales.

Kovalevskaya's mathematics are definitely too hard for most of the readers I was writing for. My idea was that it might be interesting for them to skip the mathematical sections of the book but to look at them nevertheless, because they would contain beautiful formulas and pictures. So that the book should be beautifully printed (and this was indeed the case of the French edition).

I had also two chapters on her reputation. In one of them, I examined carefully Bell's *Men of*

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3 See also [10], a beautiful book – with a highly stupid English title.

mathematics, which is partly responsible for this "bad reputation". In the other, I gathered all the "memories" I found of her, memories of friends, family, colleagues... and others.

Biographical stories, mathematics, reputation, history... I also had to take care of the fact that she was also a writer. And this I did by writing... fiction.

Fiction. In the book, there are four fiction texts. They are inspired by books of Jules Verne (who alludes to her in one of his novels), George Eliot (whom she met), A.S. Byatt (who wrote some beautiful pages about science and mathematics in some of her novels) and Italo Calvino. I also wrote a story of burned letters inspired by Georges Perec, but did not include it in the book.

Le Cosmicomiche. Remember this series of short stories [8], by Italo Calvino: each story begins with a quotation from a scientific popularization text. Then, the main character of the book, Old Qfwfq, starts to comment on this text and remembers how it was, in various (good) old times, when the Moon was so close to the Earth than we could touch it and even milk it (in *The Distance of the Moon*), or before the Big Bang (in *All at One Point*) which occurs, as everybody knows, because a woman would have liked to have enough room to cook a tagliatelle dish, and so on... Here we are, very concretely, in the "science and literature" environment. This was an important issue for Calvino but now is not the moment to develop this theme.

Back to Sofya. When she passed her thesis, Weierstrass did not want to take any risk and the thesis consisted of three papers, each of which would have been enough for a young man to become a Doktor. One was the Cauchy-Kovalevskaya theorem, another was on Abelian functions, and the third was about the shape of Saturn rings. In this paper, she proved that the section of the rings was not an ellipse. This was a continuation of Laplace's investigation. To be more precise, the mathematical model was a fluid mechanics one, namely, this was based on the fact that Saturn rings were supposed to be liquid–at that time, everybody (including Kovalevskaya and Weierstrass) knew that this was not the case: this was just an applied math problem.

Let me quote here this short story (which is not unrelated to Venice). To help identify the characters, I mention that the name of aunt M_i, reads, in French, like "Aunt Emmy", that Weierstrass' first name was Karl... and that I was very angry about Bell (the author of *Men of mathematics*).

The Rings of Saturn

Laplace showed that the cross sections of the rings of Saturn are elliptical. It was thought at the time that the rings were liquid. Today science allows that they satisfy the equilibrium conditions of a fluid, which conforms to the idea that Cassini had come up with in the seventeenth century about the rings of Saturn, that they are neither gaseous nor liquid, but that they are made up of solid particles of matter, discontinuous, separated by great distances, a multitude of little satellites linked only by their mutual attraction, very weak in comparison with that of the planet.

Not at all, yelled old Qfwfq! I remember very well that it was a liquid, a very thick liquid, like a piece of mozzarella cheese, a thick soup, like milk, yes, milk, that's what it was. If you see pieces now, it's because the milk curdled. That's not very surprising after so much time ... Anyway, at that time, it was like that, the rings were liquid. As for the cross section, it was elliptical, it's very true! You can trust me. I know them well, these rings, by dint of having them around my head.

At first we use them only to create shade, like parasols. You can't imagine what they were like, these days on Saturn, always in bright sunlight without ever a cloud. Good that they didn't last too long and that nightfall came rather soon. But nonetheless, during the day, we placed ourselves so that the rings would shelter us a bit. We didn't yet know what they were made of. But of course they were liquid. Even the first imbecile who came along could tell that.

We all were there, my aunt M_i, who made us huge plates of tagliatelle, my deaf cousin, old captain XarlraX and his two sisters, with little $S_0Ph(i)$, an agreeable company if it hadn't been for that plague of a 0-b^eLl, always turning around the plate and mostly about $S_0Ph(i)$.

How nice it was then to see little $S_0Ph(i)$ amusing herself with nothing, looking at these little pieces of wood turning, absorbed in her thoughts, counting and recounting the moons, dreaming, her eyes on the rings, covering sheets of paper with mathematical symbols, of which this plague of a $0\text{-}b^eLl$ never missed saying that it was not right for a cute girl to spend her time like that and that she'd do better by tramping around with him.

Except for watching little $S_0Ph(i)$, for filling one's heart with her joy, there was not anything to do except to admire the round arms of my aunt M_i going back and forth over the big chunks of egg dough, her white arms smeared with oil right up to the elbows. Because, for making tagliatelle, for that, there was space on Saturn. Not only space for spreading out the dough, but space for gardens or ripening tomatoes, for fields for growing wheat, for mountains for the water to come down to irrigate them, and sun for ripening the wheat, for there was no lack of sun.

What was missing were flocks and, if there were, of prairies for them to frolic and graze. "And meat?", you're going to ask me. Well, no, there was not anyone to give us any meat. Oh, we had all sorts of birds, but we didn't eat them. On other, more advanced planets there were perhaps livestock, but we on Saturn didn't have any, none at all, so we savored tagliatelle with tomato sauce and were perfectly satisfied, except perhaps that this plague of a $0\text{-}b^eLl$ was always complaining, the old grump. And it lasted forever like that until the evening, I recall that it was an evening, but of course on Saturn at that time the evenings didn't last so long, you couldn't call them long evenings, on that evening my aunt M_i exclaimed: "My children, if only I had a little milk or cream, how I would like to make you a Sicilian cassata!"

That's when little $S_0Ph(i)$, although a modest and shy girl, had an idea. A brilliant idea I can tell you. And if this plague of a $0\text{-}b^eLl$ tries to tell you that it was not she who had this idea, and that it was for example the old captain XarlraX, that would be plain meanness, don't believe it. Little $S_0Ph(i)$'s idea, it was the rings. Because, by having looked at them, she had understood, and she alone, that the rings were of milk. And because she was not lacking in practical sense, she also imagined a way of recovering some, some of this milk. "We're going to milk the rings", she said to us.

I have to tell you, the rings, they weren't so far away. They almost grazed us. So, that's how we proceeded. We climbed right to the top of the Zinc mountains, several of us went, old captain XarlraX, my deaf cousin and myself, following little $S_0Ph(i)$, who skipped along in front of us, holding her milkpot in her hands, sometimes on her head. Evidently this plague of a $0\text{-}b^eLl$ walked behind us. How unpleasant it was to have that one on our heels! This is how little $S_0Ph(i)$ proposed to realize her idea, this is how we would do the milking. We would bring a ladder, she would take off her shoes, climb it, fasten her tin milkpot to the left side, perched on the last rung, her left foot above the milkpot, yelling "I'm there!", she would manage to touch the bottom of the ring by reaching with her left arm, you can imagine that the whole thing was unstable and that our role was to hold the ladder so that she wouldn't fall. You should have seen her, little $S_0Ph(i)$, a sense of balance, a competence, a tenacity, you wouldn't have believed it, in such a pretty little girl. And pretty she was, even if she was hidden by her big hat. She had to protect herself from the sun, you can't imagine what it was like, the sun, on Saturn, at that time. When, with her left forefinger, she would reach the ring, it, by a sort of capillarity phenomenon, would begin to run gently along her arm, along her body and her left leg right until her foot, and it would fill the milkpot. When it would be completely full, she would bring it gently down and we would go back home.

And my aunt M_i not only made us a Sicilian cassata, but also some straciatella, some Neapolitan bars, coffee and vanilla ice cream with tiramisu, chocolate, nougat, rum and raisins, and even one day in a vein of exoticism, a tutti frutti. So much so that the rings started to shrink.

One milking day, little $S_0Ph(i)$ launched into a new calculation, you could see that she had been thinking about something for several days. Then she put down her pencil and said to my aunt M_i: I have to tell you, aunty, the rings have become ovoids. That is to say, she added, egg-shaped with a little part and a big part, and that my aunt M_i could understand, because we have some on Saturn, with all those birds. It must be said that for explaining something, little $S_0Ph(i)$ was the champ. Aunty, we need to stop, concluded little $S_0Ph(i)$.

Since that time, the rings have had this form. Ovoids, as little $S_0Ph(i)$ said! And since that time, aunt M_i hasn't made ices for us. We have dispersed. Now when I feel like eating a Sicilian cassata, I go buy one at Nico's, on the Zattere. I've happened to run into that plague of a

0-b^{e}Ll there, but I pretend not to recognize him. It could be that one time or another I've run across little S$_0$Ph(i), but I've never seen her, whether because I had bent down to tie one of my shoelaces or that I've turned my head to watch a pigeon fly off or that I've started running because my vaporetto has arrived.

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(Translation from the French
by Lester Senechal in [XX])

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And then?

It is not enough to write a book, then you need to make it meet its readers... Such books do not fit easily in the publishers' compatrmenes: too much mathematics, too much culture... in which box can we put this book? Moreover, the "nonfiction" concept is unknown to French publishers... I eventually succeeded in finding a publisher who enthusiastically accepted the book and made quite a good job in producing it. Unfortunately, the diffusion was not at the expected level. The English translation does not have the same quality, and it will never reach a non-mathematical readership...

It got even worse with another text that originated from this one, in which I decided to go back to the 1871 Paris Commune and to investigate the history of the relationship between the Academy of sciences and the Commune. Although this was a precise history text, with all sources quoted, this was both too literary and too scientific, not written in the expected academic way and was rejected by about twenty publishers. This one became an online publication, *Mai quai Conti* [3].

Well, I am still trying. Despite the global freezing of the publishing houses. My next book might be *La formule de Stokes, roman* (*The Stokes formula, a novel*): it found a publisher, Cassini, who wants to publish "scientific culture" books, the next problem is the diffusion... Fortunately, I also write texts which look like more standard literature, like [5] and [6] (see my article *Brief lives*, in the same volume).

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