Paulette Libermann, 1919–2007

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Paulette Libermann (© Mme Mounier-Veil)

Born on 14 November 1919, Paulette Libermann died on 10 July 2007.

In 1938, she joined the École Normale Supérieure de Jeunes Filles, also called the École de Sèvres. Until then, this school had only prepared its students to pass the (feminine) 'agrégation', which would allow them to become teachers in secondary schools (for girls). The director Eugénie Cotton (both a physicist and a left-wing militant) decided to raise the level of her students to make them attain the same level as the men of the École Normale Supérieure. Among the professors who taught these young ladies were Élie Cartan and two younger mathematicians Lichnerowicz and Jacqueline Ferrand.

At this point, Paulette Libermann's story meets History. In the fall of 1940, she was beginning to prepare for the agrégation when the so-called French State, anticipating the desire of German occupying forces, established a series of laws called the 'status of Jews'. These laws forbid the people they defined as Jews (among them Paulette Libermann) from practising a certain number of professions, one of which was teaching. Therefore she could not pass the aggregation … and this led to Élie Cartan suggesting that she start research instead.

Later, she said that the anti-Semitic laws were a bit of luck for her. But at the time, the threat to French Jews was becoming more and more serious and in 1942 Paulette Libermann's family left Paris for Lyon to live a semi-clandestine life until the liberation.

She then went back to the École de Sèvres to pass the agrégation. As this was the case for all young French mathematicians (both men and women) she was sent to teach in a secondary school. But now she knew that she wanted to do mathematical research. At this point Élie Cartan gave her a second piece of good advice, namely to start a thesis with Ehresmann.

Student of Ehresmann

It is hard to imagine what Ehresmann's school of differential geometry and topology was at that time. Charles Ehresmann himself, a member of Bourbali, passed a thesis with Élie Cartan in 1934. He had many students. The first one, Jacques Feldbau, proved in 1939 that a bundle over a simplex is a product before inventing, jointly with Ehresmann, the notion of an associate bundle … and the exact homotopy sequence of a fibration. Regrettably, Feldbau was less lucky than Paulette Libermann; the anti-Semitic policy sent him to his death in a concentration camp. Georges Reeb (1920–1993), Wu Wen-Tsun, Paulette Libermann, André Haefliger, Valentin Poenaru, are among the best known of Ehresmann's students.

Equivalence problems

Paulette Libermann published numerous papers on differential geometry. She defended her thesis, *Sur le problème d'équivalence de certaines structures infinitésimales*, in 1953. The equivalence problem is a general problem and has been investigated by many including Élie Cartan. Roughly speaking, the question is to classify, up to local isomorphism, structures on manifolds. For instance, all the manifolds of the same dimension are equivalent (this is a local question). But this is not true anymore if the manifolds are endowed with Riemannian metrics: a (curved) sphere is not locally isometric to a (flat) plane.

The metric can be replaced by a lot of structures, for instance a family of 1-forms (a Pfaff system), a notion on which Paulette Libermann worked a lot.

Symplectic geometry

Her results on symplectic geometry have become classics in the 70s and the 80s (when symplectic geometry became fashionable). The symplectic equivalence problem is solved by the Darboux theorem: all symplectic manifolds of the same dimension are locally isomorphic. There is no local invariant in symplectic geometry!

So:

(1) Look for global invariants. One of the most powerful tools is Gromov's theory of pseudo-holomorphic curves (1985). This is based on the notion of almost complex structures on symplectic manifolds, one of the notions Paulette Libermann investigated in her thesis.

(2) Make the structure more rigid, for instance by considering two transverse Lagrangian foliations on the manifold. This problem, that she investigated in one of her papers, lies at the basis of the theory of integrable systems.

She will also be remembered as the author, jointly with Charles-Michel Marle, of one of the very first textbooks on symplectic geometry.

After her thesis, she became a professor in Rennes and then in Paris. She was very helpful to young mathematicians.

Tiny, energetic, smiling, chatty, sometimes caustic, she was also a memory of the mathematical community. She liked to speak of those who helped her, either personally or professionally: Cartan's family, Jacqueline Ferrand, Ehresmann, anonymous others … and she also liked to speak of those who did *not* help her. She participated, almost until the end, to conferences all around the world. The last time we met, in April, I was leaving for Vietnam. I cannot go, she told me, too tiring, I am getting old. She was 87.

We shall miss her.

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