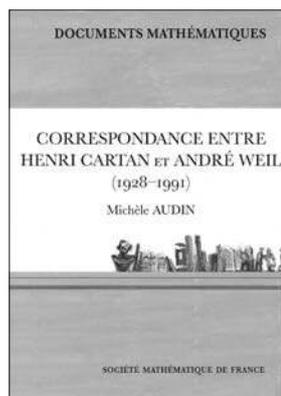


Book Reviews



Michèle Audin

**Correspondance entre
Henri Cartan et André Weil
(1928–1991)**

Société Mathématique de
France, 2010
750 pages, hardcover
ISBN: 978-2-8529-314-0

Reviewer: Javier Fresán

This book assembles more than 200 letters exchanged by Henri Cartan and André Weil from November 1928 to May 1991. Most of them were discovered a few years ago within the archives of Cartan, who does not seem to have thrown away a single paper in his life.¹ It would be hard to imagine a better editor for this correspondence than Michèle Audin, an expert, among other things, on the history of French and German mathematicians during the World Wars and the interbellum. The exquisite research she has carried out becomes evident from the first page. In particular, her extensive notes at the end of the volume are not reduced to a mere identification of the various characters and situations to which the letters refer; on the contrary, they “tell another story”, in the same way that the commentaries added by Weil to his collected works form an independent book. One can find there, just to mention a few examples: a long letter in which a very young Weil displays all his mathematical knowledge; a chronology of the Cartan seminar through Serre’s memories; and a thorough reconstruction of the anticommunism hysteria surrounding the ICM 1950, which part of the French delegation was planning to boycott if Hadamard and Schwartz did not get their visas in time.² Several documents from the recently declassified files of Bourbaki have also been included.

Let it be said from the beginning that this correspondence is quite different in style from the one maintained, partly at the same time, by Grothendieck and Serre,³ of which it could be reminiscent at first sight. While the main topic is of course mathematics, it is not the only one: as Cartan and Weil were close friends and founding fathers of Bourbaki, many letters address practical problems regarding the organisation of the group and questions of a more personal nature (such as family holidays, health issues and music). A particularly sad leitmotif is Weil’s recurring desire to find an academic position in France,⁴ for instance when Lebesgue retired from his chair at the Collège de France. Despite the great deal of time and energy Cartan devoted to supporting his friend, all his attempts were frustrated by political resentment and the

direct application of Weil’s own law, “first-rate people attract other first-rate people but second-rate people tend to hire third-raters and third-rate people fifth-raters” (p. 621). Back to mathematics, it is thrilling to discover how the slowly emerging notion of “cohomologie à domaines de coefficients variables” (p. 142) had already led, in the late 1940s, to a perfectly modern definition of spectral sequences (pp. 246–247). Young and not-so-young readers will probably smile at how breakthroughs such as the Steenrod operations (p. 217) and the Kodaira embedding theorem “toute variété de Hodge est variété projective” (p. 346) were disseminated before the arXiv!

Bourbaki

It could be seen as disappointing not to find any scoop here on the birth of Bourbaki. But this is not surprising: at that time, Cartan and Weil were colleagues at the University of Strasbourg so why should they exchange letters when they could speak in person? The first time Bourbaki is mentioned, on 29 May 1939, is just to say, “Bourbaki devient très populaire par tout: à Cambridge il est à présent le mathématicien dont on parle de plus. Il m’est revenu que Chevalley a fait une grosse propagande à Princeton” (p. 33). This shows that it was not yet the secret society it was going to become in the following years; in contrast, Weil was angry to learn on 4 May 1955 that Saunders Mac Lane had delivered a public speech at New York University in which he described himself as a “fellow-traveller” of Bourbaki (p. 365). Thanks to the letters, other elements of the legend can be put into historical context. For instance, one confirms that retirement at 50 was not a rule until the moment that Weil reached this age and wrote to Cartan, “le meilleur service que les membres fondateurs puissent actuellement rendre à Bourbaki est de disparaître progressivement mais dans un temps fini” (p. 382). If something is to be taken from the correspondence, it is that our protagonists always had Bourbaki in mind. Three early letters show Weil’s insistence on replacing the term “ensemble vasculaire” by “ordonné filtrant” (pp. 39, 45, 47). Far from being an exception, that was the general trend. Even the smallest typographical details were discussed at length; nevertheless, Weil was not unaware of the risks of this way of working, as the following extract from Bourbaki’s bulletin *La Tribu* shows: “nous ne pouvons continuer à perdre tous notre temps sur des brouilles. Lorsque le contenu

¹ Moreover, he was ready to complain to the postal service whenever necessary (p. 663).

² In Cartan’s own words: “Je crois que la seule chose que nous, mathématiciens, pouvons faire, c’est de tenter de faire déplacer le Congrès; et si on y arrive, ce sera déjà beaucoup. Mais il faut que nous fassions tout ce qui est en notre pouvoir dans ce sens, sinon nous serons aussi coupables, sur le plan de la collaboration internationale, que les Allemands qui ont admis la dictature hitlérienne.” (p. 265).

³ *Grothendieck-Serre correspondence*, edited by Pierre Colmez and Jean-Pierre Serre, AMS, 2004.

⁴ As Weil says on 26 August 1946, “Bien entendu, les USA me dégoutent, et je n’y retournerai que contraint et forcé” (p. 130).

d'un chapitre devient stable, plus n'est besoin d'un congrès plénier pour en discuter les détails" (p. 597). Taking into account the method, the scarcity of paper and the slowness of postal service, it can only be regarded as a miracle that Bourbaki survived during the war. A letter not to be missed, dated 19 July 1946, is the one in which Cartan suggests, following Chevalley, that modules could be expelled from Bourbaki's *Algebra*: "Si l'on se borne aux espaces vectoriels, l'exposition est beaucoup plus esthétique, on évite incontestablement des lourdeurs, et on facilite la tâche de la majorité de lecteurs qui, évidemment, ne s'intéresseront qu'aux espaces vectoriels. Il va sans dire que ce sacrifice ne peut être consenti que si l'intérêt des modules, dans la suite de l'Algèbre, doit être suffisamment limité pour qu'on puisse, sinon s'en passer tout à fait, du moins les reléguer à l'endroit précis où on en aura besoin" (p. 114); it follows a choleric five-page answer by Weil which definitively closed the issue.

The Weil conjectures

Another set of letters concerns the proof of the Riemann hypothesis for curves over finite fields during the Spring of 1940. In those days, Weil was imprisoned in Rouen after what he would later call "a disagreement with the French authorities on the subject of my military obligations".⁵ He did not waste this opportunity to work "sans souci extérieur", as Cartan put it (p. 63): besides proof-reading his first book and reconstructing a report on integration for Bourbaki, which had been confiscated by the Finnish police, Weil continued thinking about zeta functions. On 26 March, he writes to Cartan, "je crois toucher à des résultats très importants sur la fonction ζ des corps de fonctions algébriques" (p. 70). He then insists on the urgency of getting the answer to a question he has already asked his friend: "What is the number of n -torsion points of the Jacobian of a curve of genus g over a finite field?" This was needed for the "important" lemma on which his whole argument to prove the Riemann hypothesis relied. On 8 April, the same day that he wrote an illuminating letter to his sister,⁶ Weil announced to Cartan that he had submitted a note to the *Académie des sciences*: "Chose plus sérieuse, j'ai expédié la note sans attendre

d'avoir démontré le lemme fondamental; mais j'y vois assez clair à présent sur ces questions pour en prendre le risque. Jamais je n'ai rien écrit, et je n'ai presque jamais rien vu, qui atteigne un aussi haut degré de concentration que cette note. Hasse n'a plus qu'à se pendre, car j'y résous (sous réserve de mon lemme) tous les principaux problèmes de la théorie" (p. 79). As Weil imagined, German mathematicians did not take long to react, initiating a true "war of reviews";⁷ however, the correspondence gives no clue about his feelings regarding the accusation of "unfair play". In 1942, Weil already knew how to prove the lemma⁸ but the complete argument would only be published "eight years and more than five hundred pages later"; some letters (starting at p. 97) treat this unusual delay, which is partly due to Weil's refusal to split one of his memoirs into several articles. Somewhat more surprisingly, no mention is to be found in the remaining correspondence either to Weil's paper *Number of solutions of equations in finite fields* or to the long-range programme culminating in the proof of the conjectures stated there.⁹ To remedy this, Audin has included a fascinating letter from Weil to Delsarte, dated 13 September 1948, in which he sketches the proof of his conjectures for Fermat hypersurfaces and relates the Ramanujan conjecture to this circle of ideas (pp. 590–592).

Algebraic topology

A less expected chapter of the correspondence deals with ideas on topology and complex analysis around the invention of sheaf theory. Let us recall that Cartan was the first person to unravel the obscure papers by "l'illustre Leray" and to embark, through his seminar, the new, brilliant generation upon the search for applications. On his side, Weil was perfectly up to date with the progress on topology, as this was the field he had chosen to collaborate with the recently created *Mathematical Reviews*. Of course, the correspondence contains the already published letter¹⁰ in which Weil explains how to prove De Rham's theorem on duality between singular chains and differential forms; but this is now completed with a second letter in the same vein. Cartan's manuscript margin notes show that he had studied both texts in detail: in particular, he asks how to define, in the topological setting, "l'anneau de cohomologie (i.e. l'opération de produit)" (p. 142), which should correspond to the wedge product of differential forms. This was at the origin of his theory of "carapaces", an alternative to Leray's "couvertures", which appears on stage for the first time on 5 February 1947. Naturally reserved, Cartan was really enthusiastic about the power of this new notion: "En y réfléchissant, tu apercevas peu à peu toi même la portée de cette nouvelle théorie, qui englobe, en les simplifiant considérablement, tous les aspects connus, en apparence si divergents, de la topologie algébrique." (p. 160). Even if Weil remained sceptical for a long time, this did not stop him from encouraging Cartan to pursue his research. Another interesting exchange (from p. 311 on) was intended to help his friend prepare his ICM talk *Problèmes globaux dans la théorie des fonctions analytiques de plusieurs variables complexes*; several letters concern

⁵ *Œuvres scientifiques* vol. I, p. 547. The notes to the correspondence add many details to Weil's own account of his draft evasion: let us just mention Audin's beautiful defence of his position (p. 482–483) and the three-page letter he wrote to the Director of the New School Herbert Solow (p. 509–512).

⁶ A. Weil, "A 1940 Letter of André Weil on Analogy in Mathematics", translated by Martin H. Krieger, *Notices of the AMS* 52 (2005), 334–341.

⁷ See M. Audin, "La guerre des recensions (autour d'une note d'André Weil en 1940)", arxiv:1109.5230.

⁸ This is clear from the letter he wrote to Artin on 10 July 1942; see *Œuvres scientifiques* vol. I, pp. 280–298.

⁹ In fact, Weil only refers to Grothendieck twice: the first time to ask Cartan to give him an offprint (p. 380) and the second one in these terms: "je termine la 2e édition des Foundations (je suppose que Grothendieck ne manquerait pas de dire à ce sujet: énergie admirable, digne d'une meilleure cause)" (p. 393).

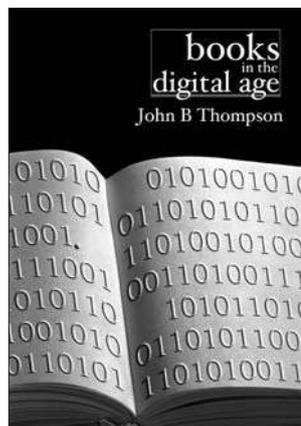
¹⁰ *Œuvres scientifiques* vol. II, pp. 45–47.

the second Cousin problem and the difference between topological and analytically trivial fiber bundles.

Needless to say, this precious document deserves much more careful analysis. Just to mention an aspect not treated in the preceding sections, the beautiful letter dated 15 June 1984 leaves no doubt as to how highly Weil thought

of his friend's father Élie Cartan, one of the secondary characters of the correspondence. My only aim here has been to draw attention to some of the passages I liked the most. Find your own!

Javier Fresán
Université Paris 13
javierfresan@hotmail.com



John B. Thompson

Books in the Digital Age

The Transformation of Academic and Higher Education Publishing in Britain and the United States

Polity, Cambridge 2005
Paperback, 480 pages
ISBN 978-0-74563-478-4

Reviewer: Manfred Karbe

The author of this book is a professor of sociology at Cambridge University and co-founder of Polity Press, a leading British publisher in the social sciences and humanities. On p. 189 he writes:

“The academic world has come to depend on the field of academic publishing (together with that of scholarly journals) as a principal means for the dissemination of scholarly work and as a key mechanism of professional certification, and, yet, ironically, most academics are woefully ignorant of what is happening in this field upon which so much of their own success now depends.

‘I think that academics are very, very, very sadly misinformed,’ commented one university press director. ‘I’d say that after ten years of proselytizing about this, I’ve made zero inroads.’ This director had her own theory of why academics were so ill-informed about the real conditions of academic publishing: because so much of their own self-esteem is wrapped up in their scholarly work, they tend to share only the success stories with their colleagues. ... Whether or not her theory is correct, it is undoubtedly the case that most academics understand very little about the real conditions of academic publishing and how they have changed in recent decades.”

This extract is one of many insights and conclusions reached through more than 230 interviews carried out over a period of three years with staff employed at all levels by 16 unidentified academic and higher education publishers in the UK and North America.

From these sources a first draft was written and comments solicited from twelve senior members in the publishing profession. The result is the first systematic in-depth study in many years of all aspects of scholarly book publishing. While the author concentrates on book publishing in the social sciences and humanities, his analysis applies, by and large, to STM (scientific, technical, and medical) publishing as well.

The book consists of four parts. It starts with an introduction of 80 pages about the business of publishing in general, which is followed by two other parts of about 100 pages each on academic publishing and higher education publishing, that is, publication of textbooks that are used as teaching material in courses at colleges and universities, from first-year undergraduate to postgraduate level. The final part, about 140 pages, is on “the digital revolution”.

Textbook publishing, as a result of conglomeratization through mergers and acquisitions, is a wholly corporate enterprise, with Pearson, Thomson and McGraw-Hill the dominant players accounting for 73% of the U.S. college market in 2002 (p. 204). Academic publishing presents a more diversified picture involving participation of a large number of university, non-corporatized and non-profit publishing companies, mostly with output of research in the form of books which range from high-level monographs and proceedings to books written for a broader readership. Here a dramatic change has taken place since the mid-1980s, which is usually referred to as the so-called “crisis of the monograph” also widely known as “death of the book”: During the 1970s academic publishers could comfortably expect to sell 2,500 hardback copies of a scholarly monograph; today many of them must accommodate to total sales as low as 400–500 copies worldwide (pp. 93–94). Three reasons are identified for this: firstly, the squeezing of higher education budgets in general, and library budgets in particular, from the early 1970s onward (p. 98); secondly, higher expenditures for periodicals caused by both a steep rise in cost of journal subscriptions and growth of volume (especially in the STM fields but also in the humanities); thirdly, growing investment in IT services (p. 99). Special attention is given to the role of consolidation in journal publishing, which is elaborated on the example of Elsevier (p. 100–101). In recent years much heated debate has been generated over the impact of a small number of publishers having emerged as the key players, controlling a large proportion of journal titles, putting “them in a position of considerable strength when it comes to de-