Fatou, Julia, Montel, The Great Prize of Mathematical Sciences of 1918, and Beyond

by Michèle Audin

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1915 The war in Europe is raging furiously. Millions of soldiers died in its first months. Among those who fought valiantly, and were gravely injured, was a former student of the Ecole Normale Supérieure. He had entered the School in 1911 and was one of the brightest students ever seen, ranking first in the entrance competition both at the Ecole Polytechnique and the Ecole Normale Supérieure, showing promise of becoming an important mathematician. In 1915 and during the following years, while undergoing intensive medical treatment—including multiple surgeries—at the military hospital of Val de Grâce, he wrote hundreds of pages of difficult mathematics on various subjects. This remarkable character was Gaston Julia.

Meanwhile, the French Academy of Sciences continued to meet every Monday, to receive and publish scientific contributions as "Notes aux Comptes Rendus", and to accept *plis cachetés* (sealed papers). The academy decided to launch a competition for a *Grand Prix* on a theme in mathematics that seemed ripe and promising: the investigation of global properties of iterations of rational mappings in the plane. Gabriel Koenigs, not yet a member of the Academy, had already investigated the local properties and, more at the heart of the subject, Pierre Fatou had a series of striking examples of exotic behaviour for the closure of the orbits. Fatou had published a Note in 1906 on the subject while writing his dissertation on trigonometric and Taylor series, and he certainly was considered as a natural competitor. The Grand Prix was to be awarded in 1918.

Fatou had entered the Ecole Normale Supérieure in 1898, four years after Paul Montel, Henri Lebesgue, and Paul

Langevin. Montel's most famous achievement is the introduction of normal families (*familles normales*) of analytic functions, one of the first and best examples of the importance of the notion of compactness in analysis. Both Fatou and Julia used normal families as soon as they appeared in 1917. Given a rational function R and its iterates $R^n (n = 1, 2, 3, \ldots)$ the points around which the R^n are a normal family constitute an open set; the complementary set is closed and it is called now the Julia set. Fatou had designated it by F (probably for frontier), and Julia by E', the derivative of the set E consisting of the repelling periodic points of all iterates.

1916, 1917, 1918—the war was still raging, and academic life went on. Julia defended his thesis and proceeded immediately to work on the iteration problem. Fatou, pushed by Montel, wrote his *mémoires* on the subject. On March 15, 1915, the Academy expelled Felix Klein and two other German scientists for a text they signed about the German army; Charles de la Vallée Poussin from Louvain (destroyed by the German army) was elected instead. In 1918 Fatou did not compete for the Great Prize; there were three competitors: Samuel Lattès, Salvatore Pincherle, and Gaston Julia. On November 11, 1918, Armistice Day, the General-in-Chief, Marshal Foch, was elected as member of the Académie des Sciences. On December 2, Gaston Julia received the Great Prize, his mémoire was "crowned".

Most of the book is a description of this period from different angles: the characters, the mathematics involved, and the atmosphere in France and in particular at the Academy at the time. It is a fascinating piece of work. Inside the frame that I have just described, Audin draws the portraits, the environment, the foreground, and the background, and does it with her own views, her feelings, and her passion for mathematics and for the truth. Although the period 1915–1918 is both the beginning and the crux of the book, Audin continues to explore academic life afterward—how people evolved and how mathematics went on.

Audin uses modern terms when necessary to explain the papers of Fatou or Julia, and figures not contained in the original articles. For instance, we see the common boundary of the basins of attraction of the function $\frac{1}{2}(z + z^2)$, one of the first examples considered by Fatou, a Jordan curve with no tangent line at any point. We see also the "basilica", named after the basilica St. Marco in Venice and its reflection in water viewed by Benoit Mandelbrot, as the Julia set for the function $z^2 - 1$.

Although Fatou certainly had such figures in mind, there is no trace of them. Julia also had similar figures in mind, and fortunately some of them were kept: handwritten figures were included in plis cachetés he had sent to the Academy. One of them is the Julia set of the function $\frac{1}{2}(3z - z^3)$, reproduced on the cover of the book. We can now make better figures with computers, but clearly all the analytic developments in the papers of Fatou and Julia have geometric roots and, as Audin claims, figures were drawn. There are remarks or comments of this sort throughout the book: about the terms in use (who introduced the term "réflexif", the term "Julia set"?), the relations with other theories (the Newton method for computing the roots of an equation, the Kleinian groups and their study by Poincaré), the normal families and the points J of Julia, the modern developments

(Douady, Sullivan, the Mandelbrot set), and the construction of figures (Chéritat versus Mandelbrot). There are beautiful mathematical figures in the book, mainly by Arnaud Chéritat. However, the main interest is the quality of the information and of the exposition. Even specialists in the field will find something new in the book, and nonspecialists will have an excellent introduction to modern complex dynamics. About half of the book consists of mathematics.

The collection of mathematical personages includes not only Fatou, Julia, and Montel, but Hadamard, Borel, Lebesgue, Picard, Painlevé, Darboux, Paul Lévy among the French, and Ritt, Hausdorff, Ostrowski among the foreigners. There are long and important quotations from their writings, and accurate analysis of their works and their attitudes. Audin acts as a historian, consulting archives, reports and letters, and books and journals. All people appear in their historical context, beginning with the First World War. For example, we see Picard and Painlevé greeting the new member Marshal Foch in a grandiose ceremony at the Institut de France. Painlevé was an important politician as well as an excellent mathematician, and he was president of the Academy when Julia received the Prize. His address at the time of Foch's induction is quoted at different places in the book—it expresses the patriotism of the time in a more decent form than that of some other academicians. Hadamard appears in many places, and an important report he made on Fatou is reproduced entirely. He was the person who knew Poincaré the best, and he certainly was active in the choice of the subject of the Grand Prix. Two of Hadamard's sons were killed in the First World War, the third in the Second. About Fatou he wrote that he gave up competing for the Prize for health reasons. Fatou had poor health indeed, but also he would have been conscious that the atmosphere of the time gave Julia the edge—or he simply didn't want to compete.

The *mémoires* of Julia and Fatou were published at the same time, they deal with the same subject, they worked independently, and the Academy had to decide delicate questions of priorities on the basis of published *Notes aux Comptes Rendus* and unpublished *plis cachetés*. Julia's originality was fully recognized. His career was rapid and bright. He was elected as a member of the Academy in 1934, though he had been ranked second to Paul Montel by the committee of mathematicians—Montel had to wait for a few years. Later the same situation was created in 1952 when Garnier, supported by Julia, was elected instead of Maurice Fréchet, who had been ranked first by the mathematicians. Fréchet had to wait, and Paul Lévy refused to be a candidate before Fréchet was in. The age of entrance to the Academy increased dangerously. Julia was active in many ways, in particular in the International Union of Mathematicians. Audin is critical of his position in different circumstances, and ends the book with a request for a biography of Gaston Julia. A few readers were shocked by this disrespect, and I shall discuss one of the expressions of this disrespect later.

The book also covers the relations between Julia and Montel. Apparently they were quite smooth in 1932 but turned tense later. A report by Paul Lévy in 1965 provoked an exchange of letters about the points J of a family of analytic functions compared to the irregular points of Montel—Lévy showed that these two notions defined the same points and Montel wanted his contribution to be recognized. Paul Montel appears frequently in the book and not only through the "normal families". He was more than just a decent dean of the Faculty of Sciences of Paris in a difficult period, that of the German occupation.

In 1965 Montel was 89 and, Audin observes, judging from his writing, far from senile. I can confirm this. The large amphitheater of Orsay, named now amphithéatre Henri Cartan, had just been constructed, and I heard Paul Montel speaking without microphone with a clear and powerful voice, jokingly describing his life: he had never been ill, but he had recently contracted a serious disease: *une incurable longévité*. He died in 1975, aged 98. Fatou had died in 1929, and Julia would die in 1978.

Between Julia and Fatou there had been no competition. Fatou was an astronomer, more exactly adjoint-astronomer—he was promoted to a full position as astronomer only two years before his death. He was highly appreciated by leading mathematicians, Hadamard, Lebesgue, Montel, to name a few. In his writings he was as unselfish as possible. For example, what Fatou called the Parseval relation is due to him alone. Lebesgue had to push him to write his thesis, on trigonometric and Taylor series; in his books Lebesgue mentions discoveries of Fatou (for instance, the first and best example of a trigonometric series that converges everywhere without being a Fourier-Lebesgue series). Reading Fatou is a pleasure. Comparing the *mémoires* of Fatou and Julia reveals very different personalities from the very beginning. Posterity has reestablished Fatou's position as a mathematician; Milnor has said: "The most fundamental and incisive contributions were those of Fatou himself. However, Julia was a determined competitor and tended to get more credit as a wounded war hero."

About half of the book is devoted to Fatou, with unpublished letters and testimonials. Michèle Audin investigates traces of his life by consulting the archives of the French Mathematical Society (of which he was an active member) as well as collections of letters held by his family. His works are listed in the bibliography, and I know no other place where this list can be found. His letters reveal an exceptional personality, clear and generous. There is plenty of good mathematics in these letters, including problems, and I suggested once that the book should be read starting with these letters in the Appendix.

Fatou had no students. Julia however had a late and bright student, Jacques Dixmier, and Alain Connes was a student of Dixmier. Montel was the director of many theses, at a time when the direction was purely formal. The thesis of Fatou, the normal families of Montel, and the contributions of Julia to the theory of functions quickly became classical, but the matter of iteration and the *mémoires* of Fatou and Julia remained untouched in France for almost 60 years, becoming popular only now with the explosion of complex dynamics. (In Sweden they were better known: Carleson became interested in the 1960s, and his student Hans Brolin wrote his dissertation on iteration of rational mappings in 1965). Why did it take so long in France? The

book does not answer the question but there are some facts to take into consideration. In France, the war killed half of the possible future mathematicians, and the younger generation reacted against the older one. Moreover Picard was opposed to the theory of sets and all new matters in mathematics, and he had great influence and power at the time. Iterations came back in a different way, through the works of Denjoy in the 1930s. Adrien Douady became interested in the *mémoires* of Fatou in the late 1970s. Benoit Mandelbrot had been advised by his uncle Szolem Mandelbrojt to read Fatou and Julia in the 1950s, which he did, but he turned to linguistics and other matters before rediscovering the Julia sets and describing the Mandelbrot set in the 1980s.

Michèle Audin makes the cutting comment that Julia's main contribution to mathematics, apart from his work on iteration and more generally complex analysis, may have been the way he collected money for the publication of the *Collected Works of Henri Poincaré*. This raises a serious question. The French are slow to publish the collected works of their best mathematicians. Poincaré is one exception, Julia another. Actually the last volume of the *Collected Works of Gaston Julia* contains discourses by Julia and on Julia, in particular at the occasion of his Jubilee, and it tends to give a glorious impression. It is worth reading that volume as a complement to Audin's, and to have a look at the others of a more scientific character. The *Collected Works* introduce the reader to the personality of Julia as well as to his scientific achievements. The reader is left to judge.

For Fatou and Montel this possibility doesn't exist and it is a pity. Audin has a strong sympathy for Fatou and a definite esteem for Montel. In my opinion this is fully justified, even if she strikes a blow at the glorious figure of Julia. It would be desirable to have the collected works of Montel and of Fatou for many reasons, in particular as a complement to this book.

The book is impressive in its content, the amount of information it gives, and in the quality of the investigation and of the exposition. For many readers, as for myself, the fact that the author is present on every page with a remark, a comment, and sometimes a mockery, will make it more pleasant and easy to read. In a way you discover the author as well as the characters she paints. I like her judgment and style, but I understand that they may irritate other readers. Mathematics is not out of the world, and things in the world are not a model of smoothness. To express in the same book the beauty of mathematics, the ugliness of the war, and how that interacts with the life of real people, seems to me a remarkable achievement.

Note. This report was written after reading the original version of the book, that is, the French version. The English version is more than a translation, it is enlarged and the presentation is different. It should be observed that it is the first book of a new subseries of the Springer Lecture Notes in Mathematics devoted to the history of mathematics. The creation of this new subseries is good news. I only regret that the new version contains no reference to the previous one, except an appreciation on the back of the cover. A book devoted to history should include its own history.

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