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★**Fatou, Julia, Montel, le grand prix des sciences mathématiques de 1918, et après. . . [Fatou, Julia, Montel, the Grand Prize of Mathematical Science of 1918, and later. . .]**

Springer-Verlag, Berlin, 2009. vi+276 pp. \$49.95. ISBN 978-3-642-00445-2

This book is a history of important mathematical developments during a turbulent time. In 1906, Pierre Fatou, an employee of the Paris Observatory, startled the mathematical world by describing simple examples of rational maps from the Riemann sphere to itself with strange and unexpected properties. The local theory of iterated holomorphic maps had been studied already for more than thirty years, with important results due to Ernst Schröder in Germany, Gabriel Kœnigs in France, and Lucyan Böttcher in Poland. However, the few examples of global behavior which were known were quite well behaved, and did not display the chaotic and fractal behavior that we have now come to expect.

In the Fatou examples, such as $f(z) = z^k/(z^k + 2)$, nearly every orbit

$$z \mapsto f(z) \mapsto f(f(z)) \mapsto \dots$$

converges to the attracting fixed point at the origin, but there is a small set of exceptional points for which the orbit remains bounded away from zero. These form what is now called a “Cantor set”: a compact totally disconnected set with no isolated points. (Nowadays it would seem more natural to modify this example by putting the attracting fixed point at infinity, replacing $f(z)$ by the conformally conjugate polynomial map $g(w) = 1/f(1/w) = 2w^k + 1$.)

This example aroused so much interest that in 1915, well into the First World War, the French Academy announced that it would award its 1918 *Grand Prix des Sciences Mathématiques* for work in the global study of iteration, for example in the “simplest case” of iterated rational maps.

One very determined contender for the prize was Gaston Julia, a much younger mathematician whose face had been horribly wounded early in 1915 during combat along the *Chemin des Dames*, north-east of Paris. (It would be difficult to overestimate the trauma of the war for France. As just one example, it has been estimated that 40% of French university students were killed or wounded during the war, and the fatality rate was even higher for students in the *École Normale Supérieure*, who traditionally fought in the infantry.)

The most obvious contender for the prize was Fatou himself, who was engaged in a very active study of the problem. However, for reasons which are not known, he never submitted an entry. Poor health may have played a role. Audin suggests that, most likely, he decided not to compete because he was offended by the competitive atmosphere. (Perhaps he was dissuaded by veiled hints that Julia would accuse him of stealing his unpublished work, although this was obviously not true; cf. [D. S. Alexander, *A history of complex dynamics*, Aspects Math., E24, Friedr. Vieweg, Braunschweig, 1994; [MR1260930 \(95d:01014\)](#)].) According to Audin, Fatou surely would have been awarded half the prize if he had competed, and he would then have been able to find a position as a mathematician.

There were two other contenders: Samuel Lattès, from the University of Toulouse, and Salvatore Picherle, an older mathematician from the University of Bologna. In the event, it was Julia who was awarded the prize, with a posthumous honorable mention to Lattès, who had died of typhoid fever a few months earlier.

In fact the theories worked out by Fatou and Julia are very similar, although the two mathematicians apparently never communicated with each other. Both developed extensive and beautiful results, making heavy use of Paul Montel's new theory of normal families. However, there was a drastic difference between the ways that Fatou and Julia were treated by the French mathematical establishment. Audin points out that Julia had a long and successful career, with positions at the École Polytechnique, at the École Normale Supérieure, and at the Sorbonne. He was awarded not only the grand prize in 1918, but other prizes in 1917, 1926, 1928, and 1931, was elected to the Académie des Sciences, and twice received a Peccot Prize Lectureship. His collected works have been published in six volumes. Fatou remained at the Paris Observatory, and never obtained a position as mathematician. His collected works have never been published.

One might believe from this discrepancy that Julia was a much better mathematician. However, it seems to be generally accepted today that although Julia was an important mathematician, Fatou was a substantially more important one. Fatou's expositions are clearer and more careful, and he accomplished more than Julia. He initiated the global study of iterated rational maps. His thesis, submitted to the University of Paris in 1907, proved the important result that any bounded holomorphic function on the open unit disk has radial limits in almost every direction. He later constructed the first example of a nondegenerate holomorphic map $f: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ such that the image $f(\mathbb{C}^2)$ is not dense in \mathbb{C}^2 . Audin is not alone in finding the difference between the treatment of these two quite remarkable.

The above is a brief outline of events which are described in much greater detail in Audin's rich account. She provides a blow-by-blow description of developments in the mathematical world between 1915 and 1918, as the war took its course, and follows it up with a discussion of progress in holomorphic dynamics (as well as continued mathematical disputes) until 1965. Her book is carefully documented, and provides many details about the lives and personalities of the protagonists. I recommend it highly.

Reviewed by *John Willard Milnor*

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