

Fatou and Julia sets

Introductory talk

Michèle Audin

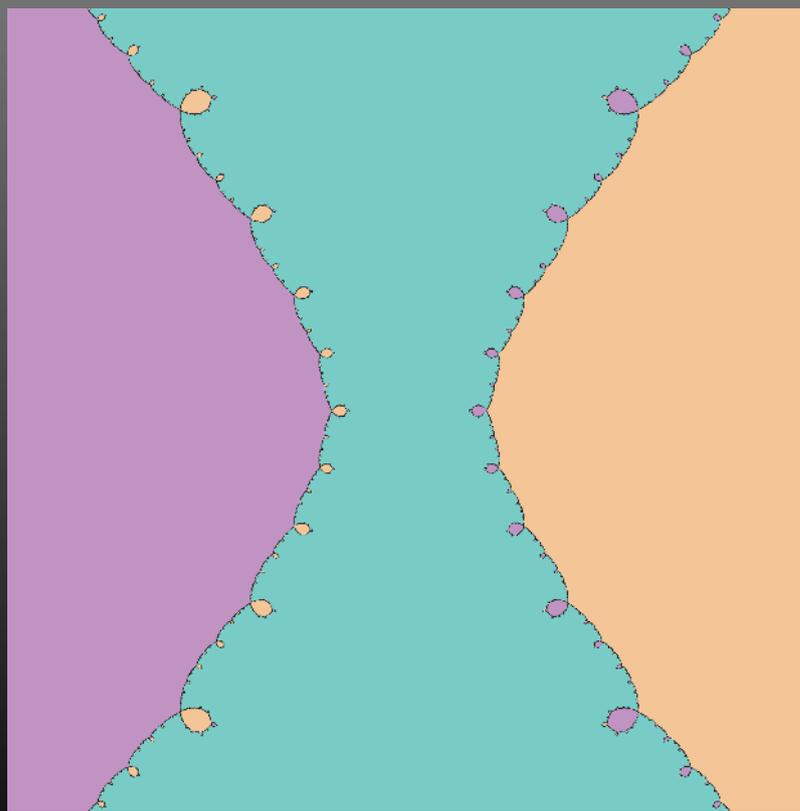
Institut de recherche mathématique avancée

CNRS et Université Louis Pasteur

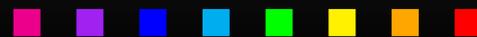
Arbeitsgemeinschaft, Oberwolfach, March 31st, 2008



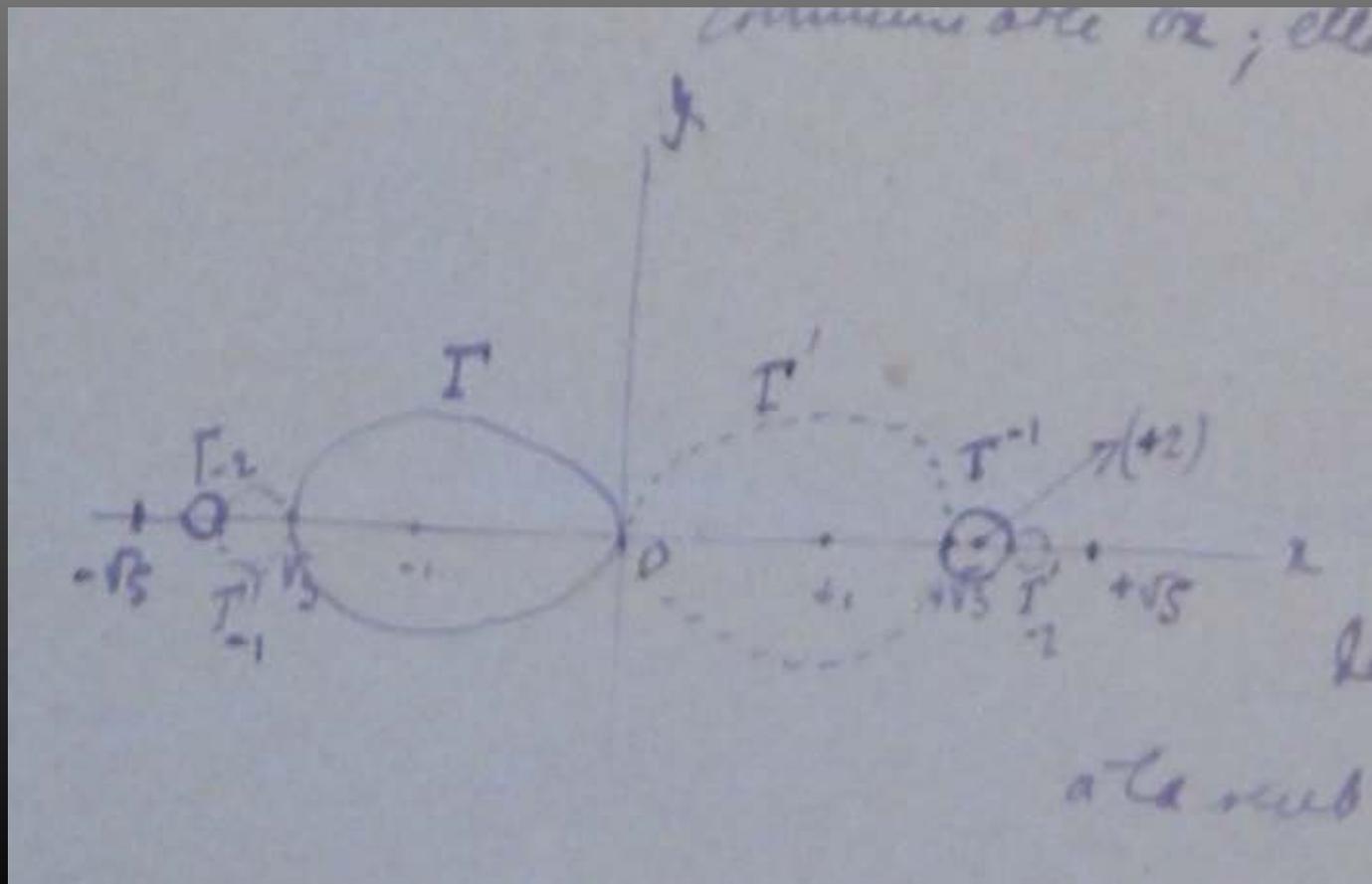
$$R(z) = \frac{2z^3}{3z^2 - 1}$$



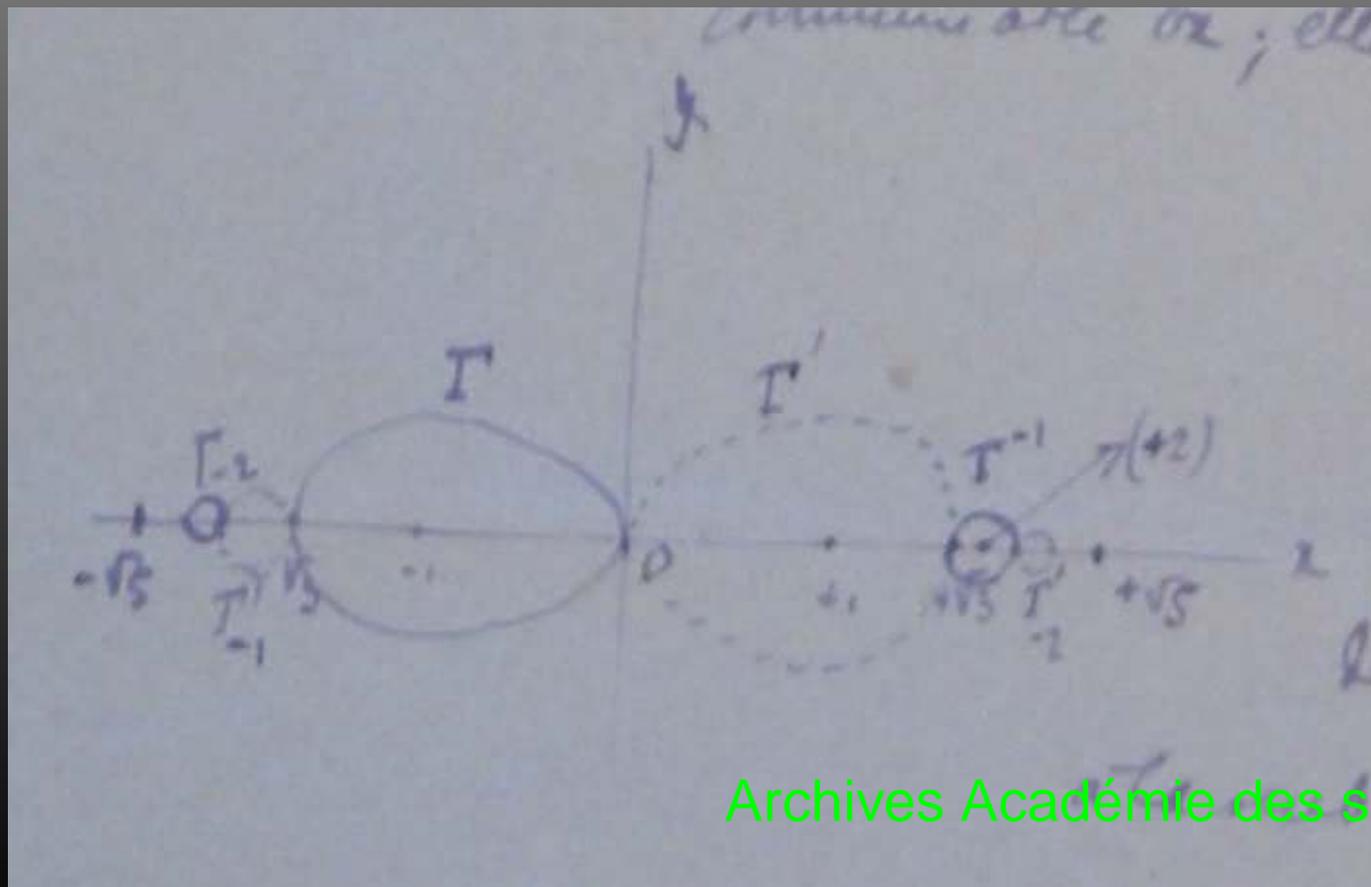
All pictures in this talk (except explicit mention of the contrary) are due to Arnaud Chéritat



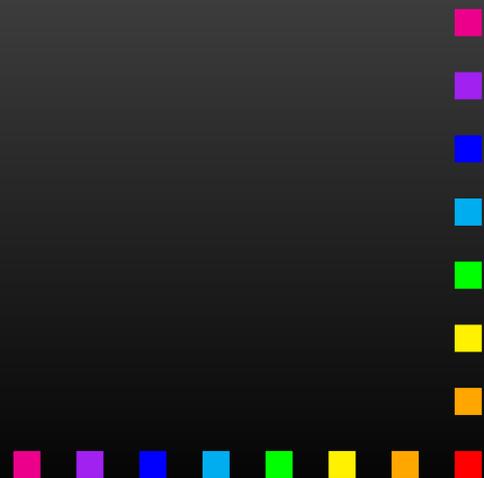
Gaston Julia, 1917



Gaston Julia, 1917

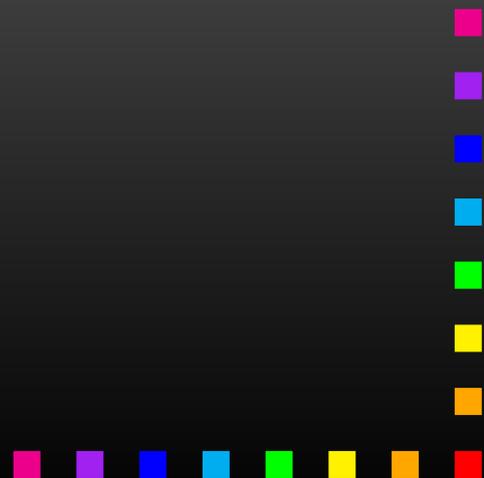


After Gaston Julia, 1918



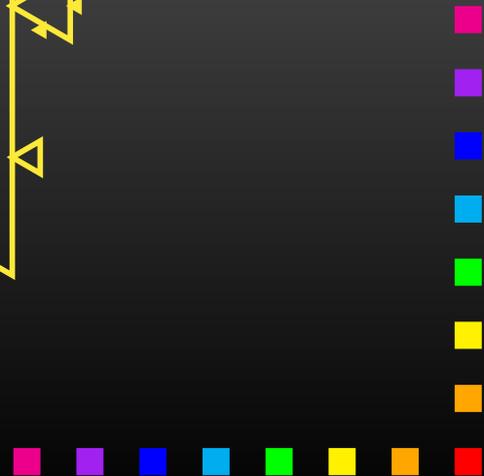
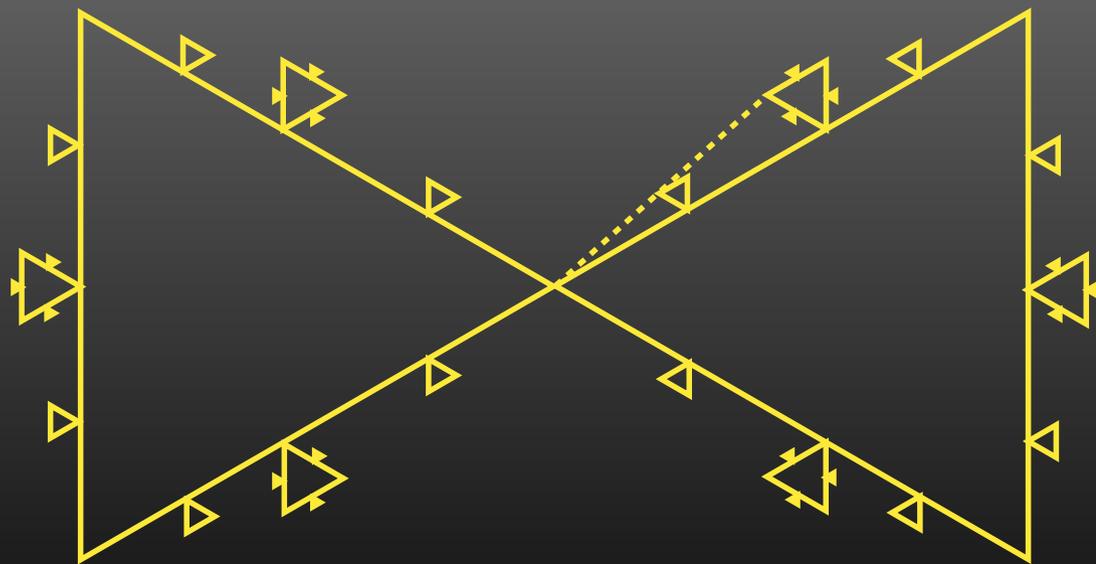
After Gaston Julia, 1918

Journal de mathématiques pures et appliquées,
1918



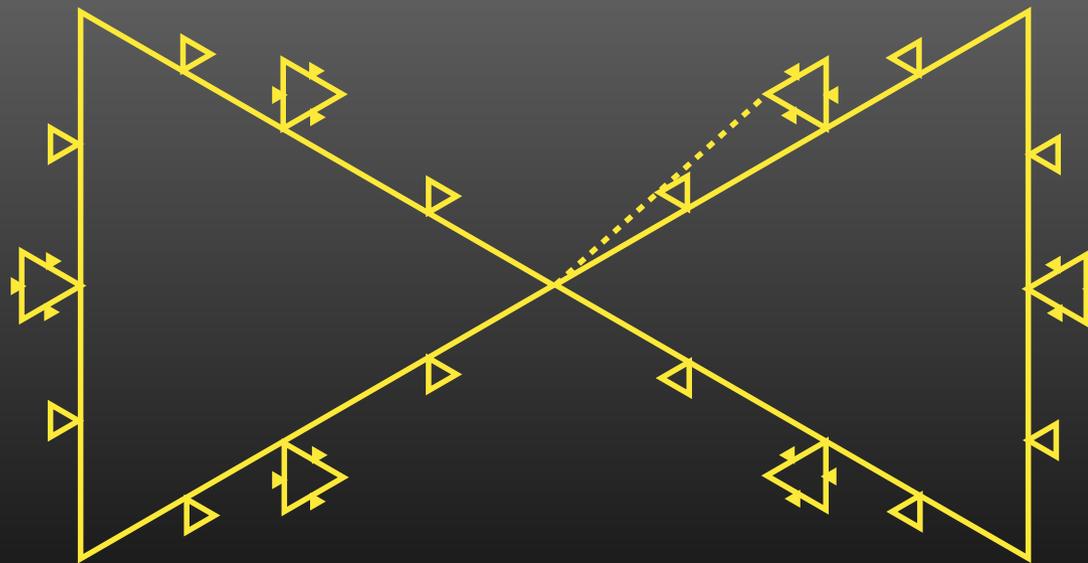
After Gaston Julia, 1918

Journal de mathématiques pures et appliquées,
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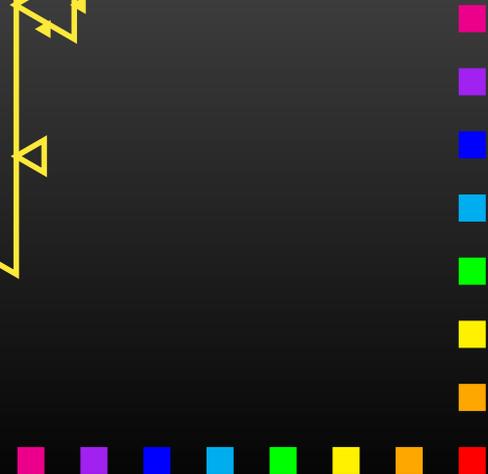


After Gaston Julia, 1918

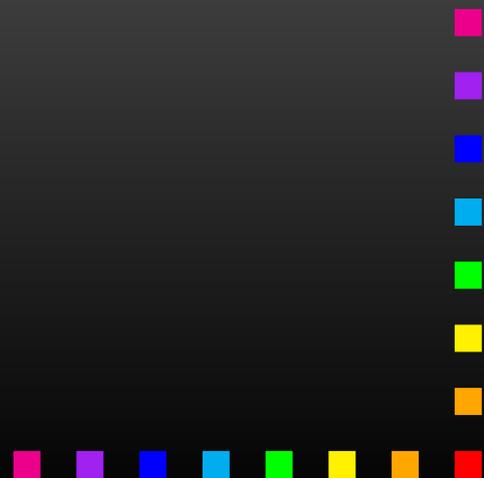
Journal de mathématiques pures et appliquées,
1918



Inspired by von Koch (1906)



Pierre Fatou, 1919



Pierre Fatou, 1919

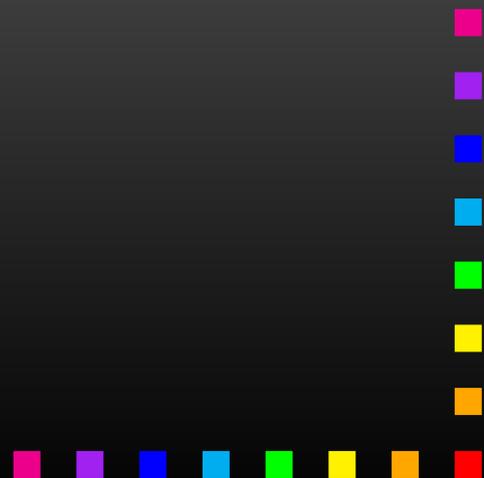
$z = 0$). On peut se faire une idée de l'ensemble \mathcal{J} par l'exemple suivant qui est analogue : considérons deux circonférences tangentes entre elles extérieurement; traçons ensuite deux autres circonférences tangentes extérieurement respectivement aux deux premières et complètement extérieures l'une à l'autre; traçons ensuite quatre circonférences tangentes extérieurement respectivement aux quatre circonférences déjà tracées, mais extérieures les unes aux autres et ainsi de suite de manière que les rayons de ces circonférences successivement tracées tendent vers zéro et que chaque point de l'une d'elles soit limite de points de contact; l'ensemble de toutes ces circonférences et de leurs points limites forme une courbe ayant une infinité de points doubles qui est analogue à notre ensemble \mathcal{J} ; les circonférences doivent seulement être remplacées par des courbes de Jordan sans points doubles qui, comme nous le verrons au Chapitre suivant, n'ont de tangentes en aucun point.

Bulletin Soc. Math. France



Pierre Fatou, 1919

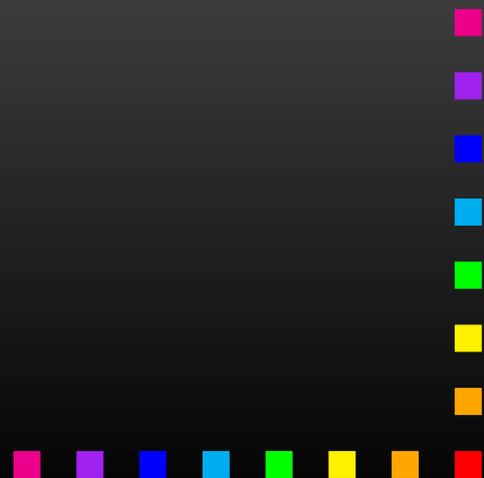
In English:

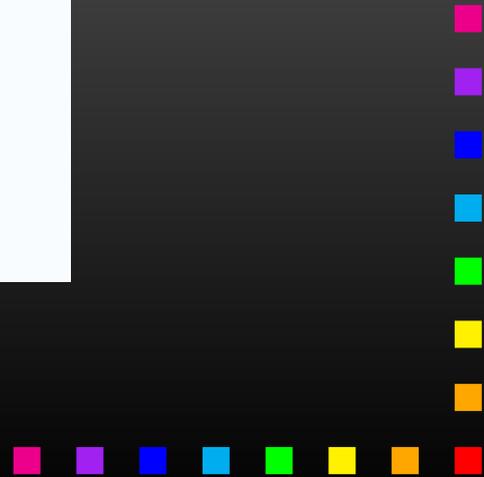
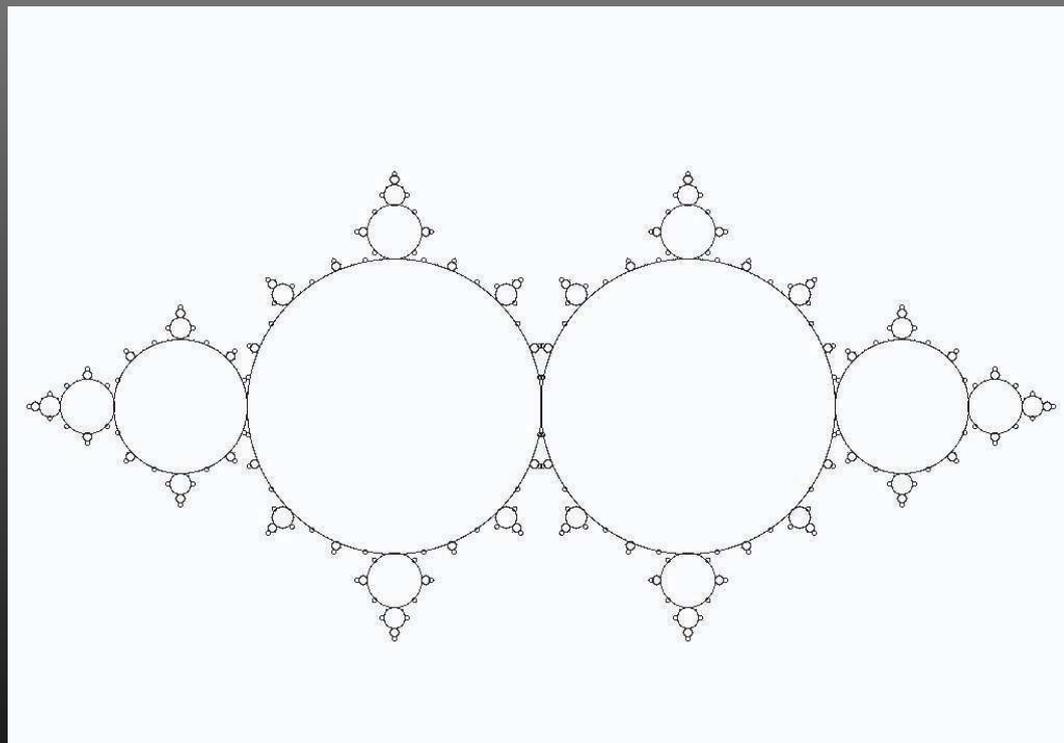


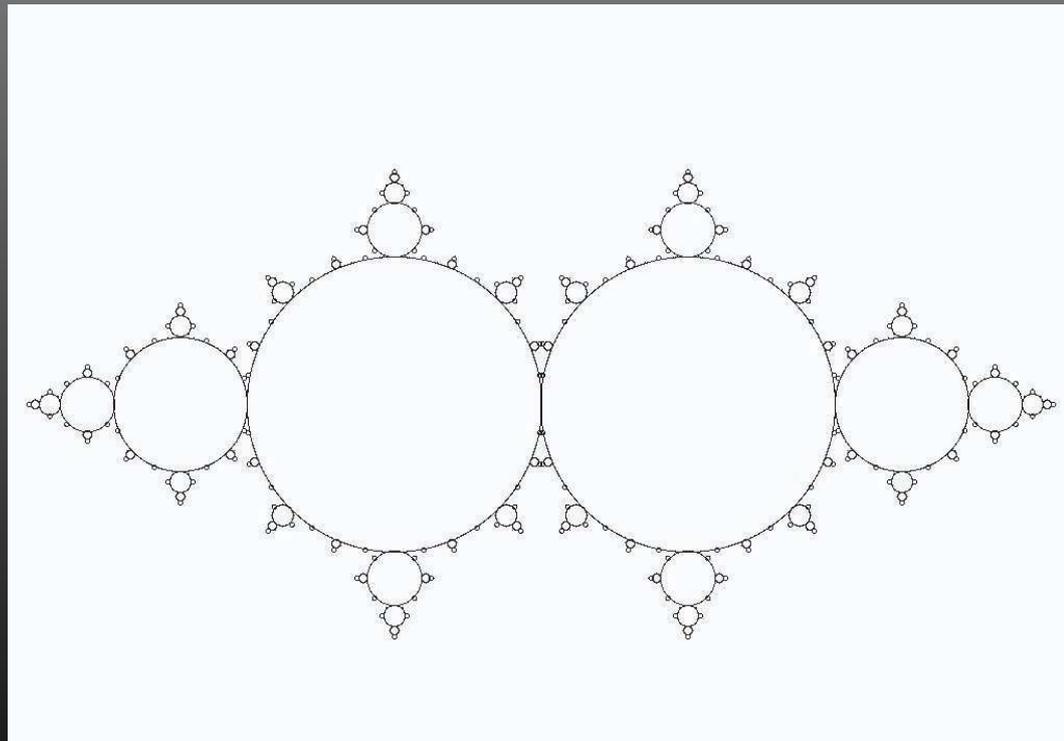
Pierre Fatou, 1919

In English:

Use tangent circles

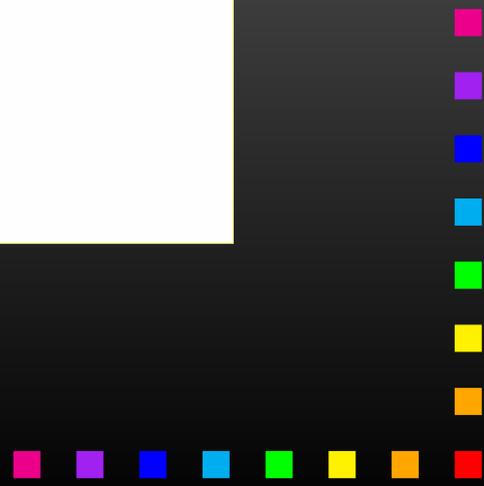
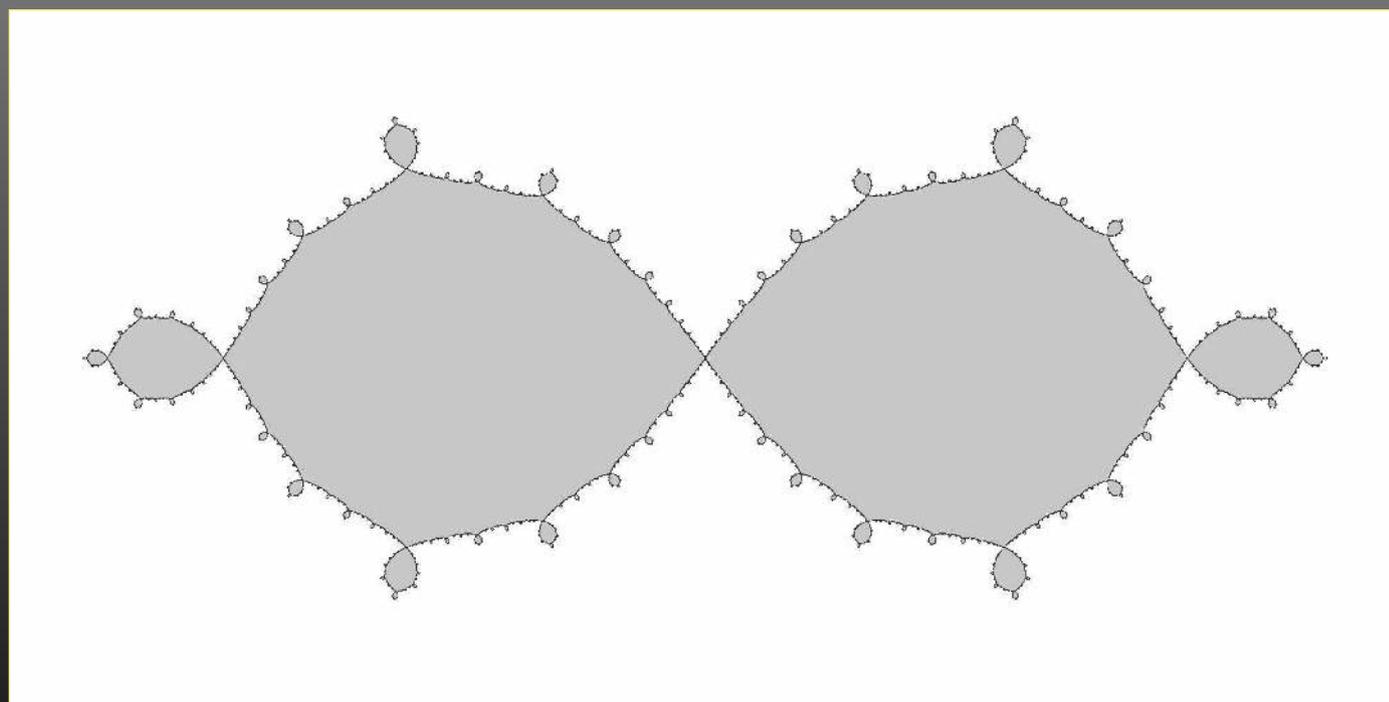


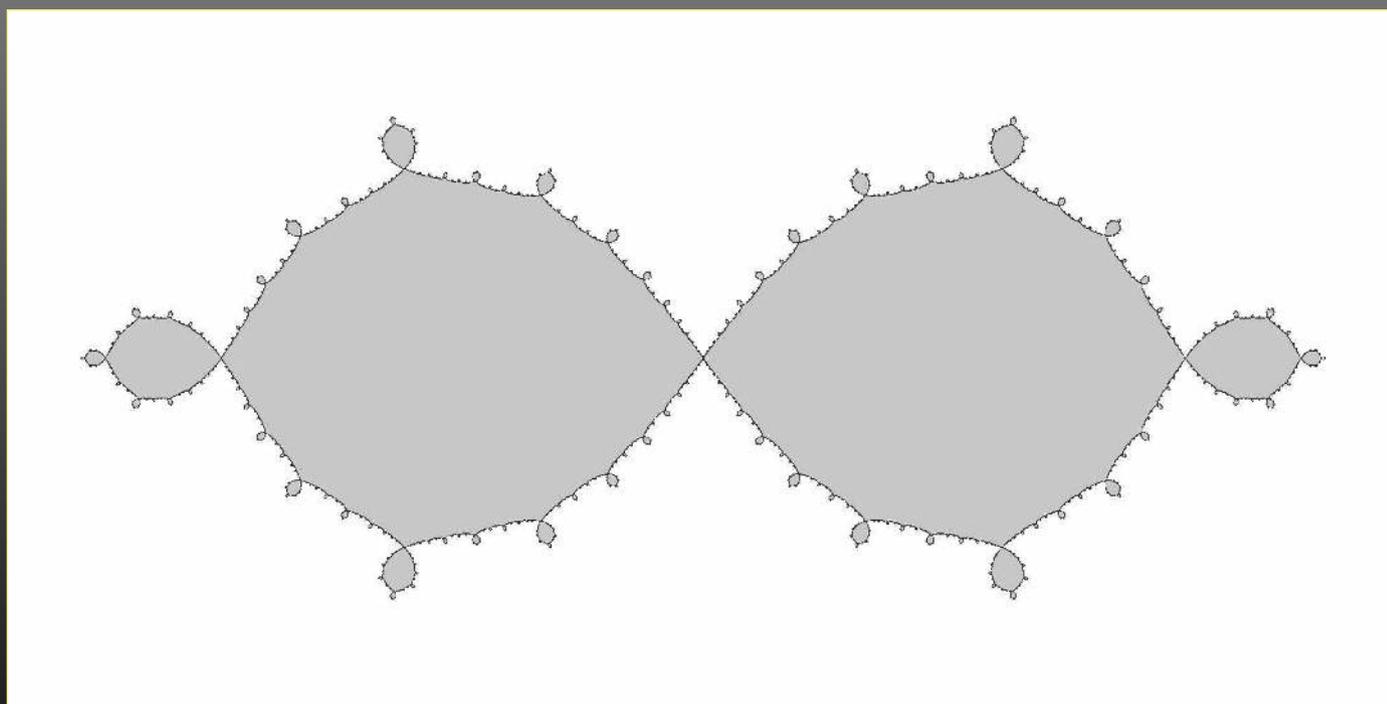




Norbert Steinmetz, synthetic Julia set, *Rational iteration*, page 135

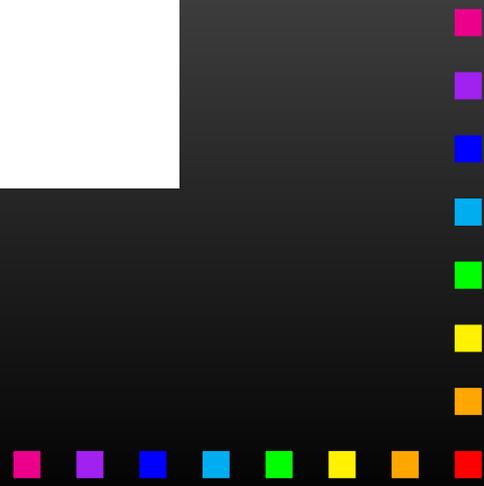
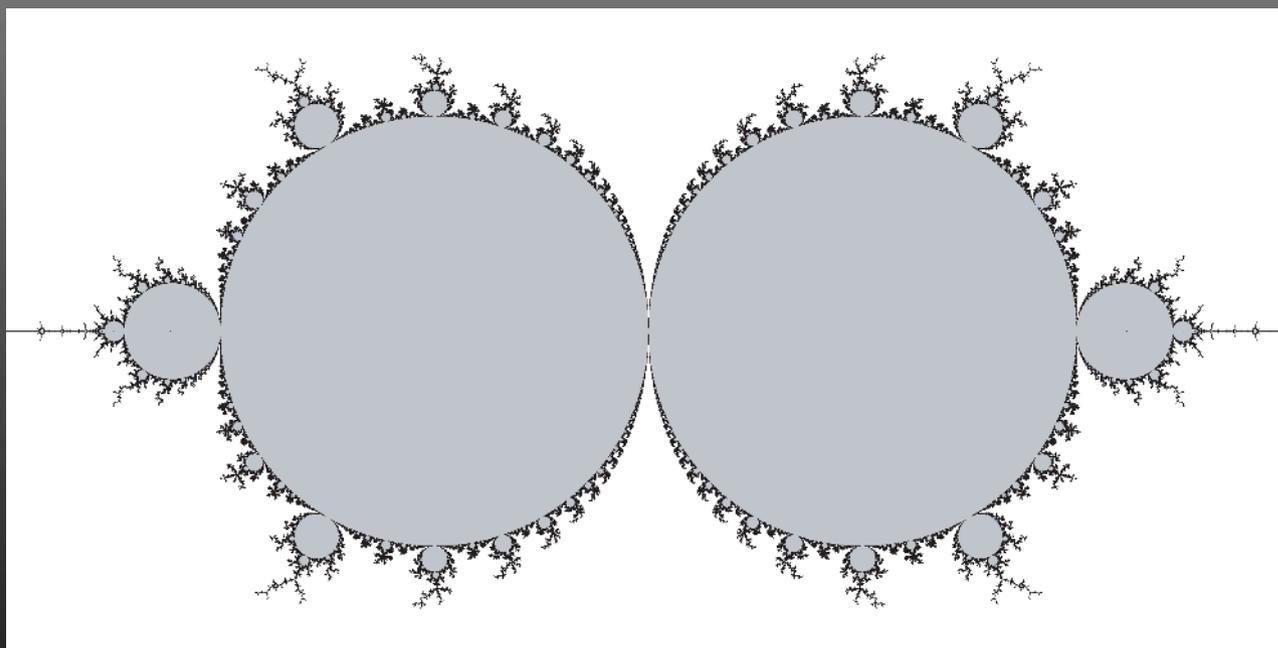


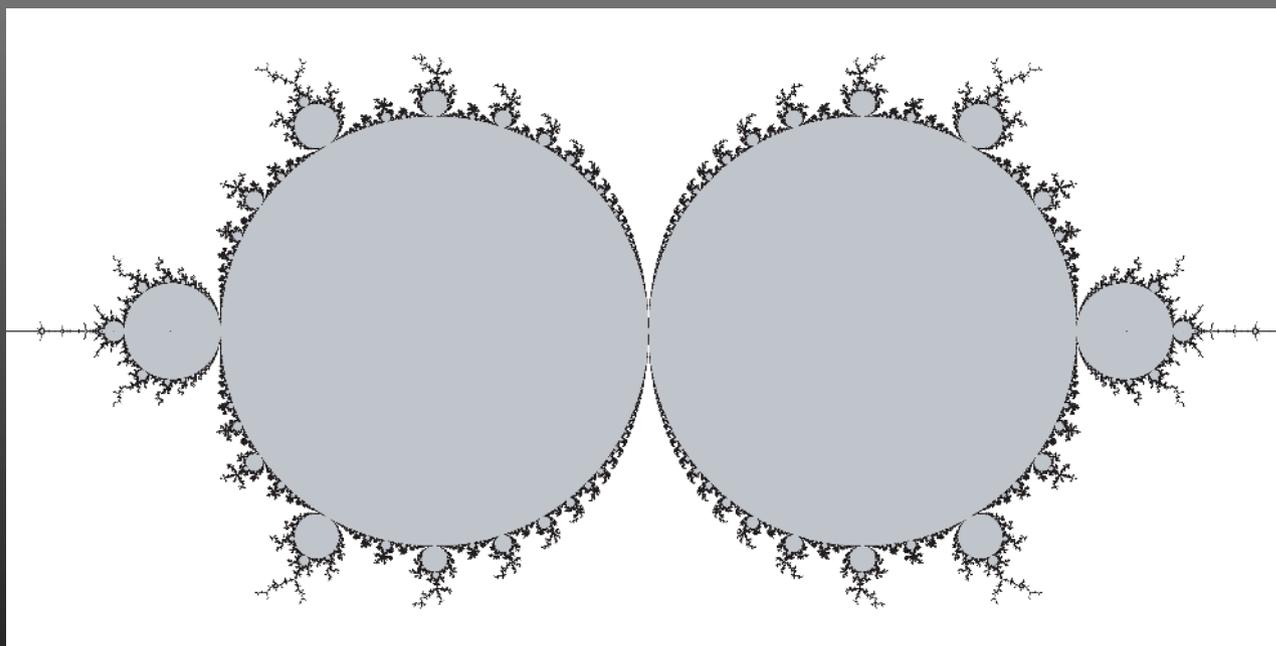




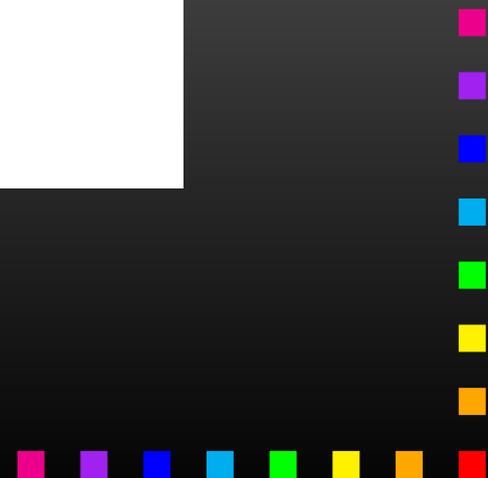
Actual Julia set, Arnaud Chéritat and his computer



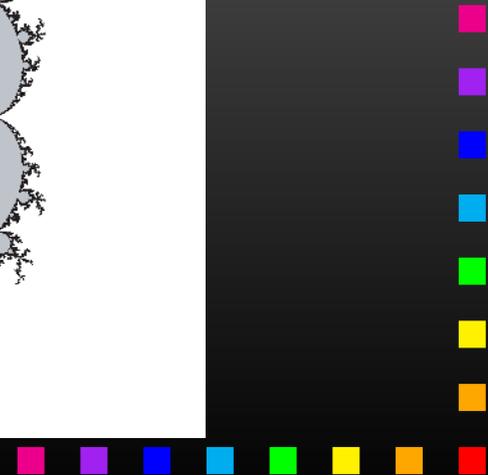
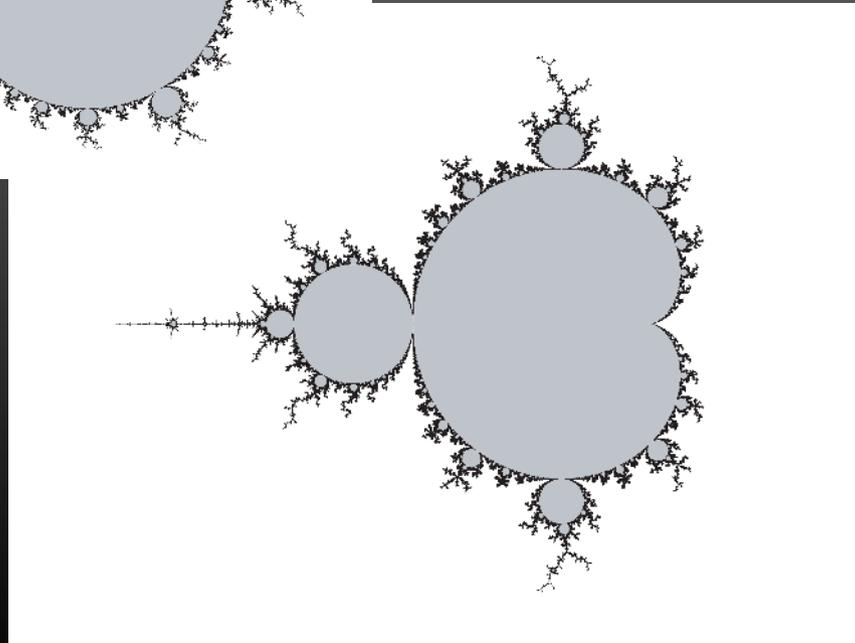
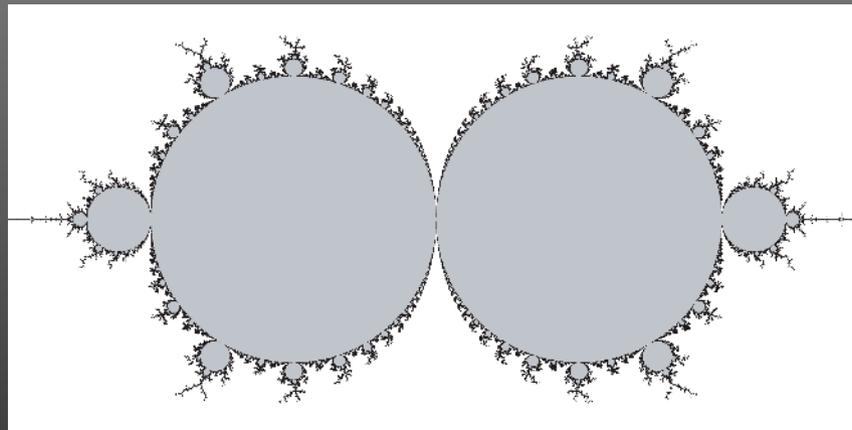


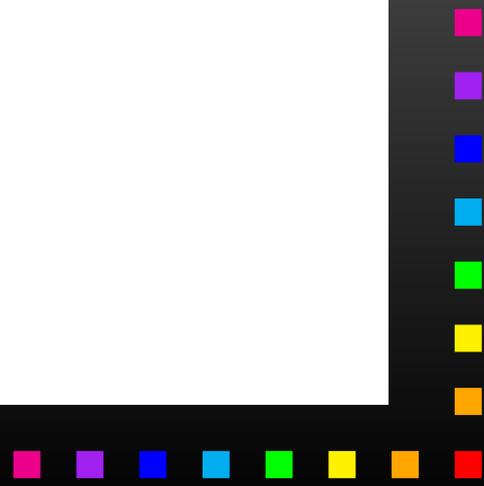
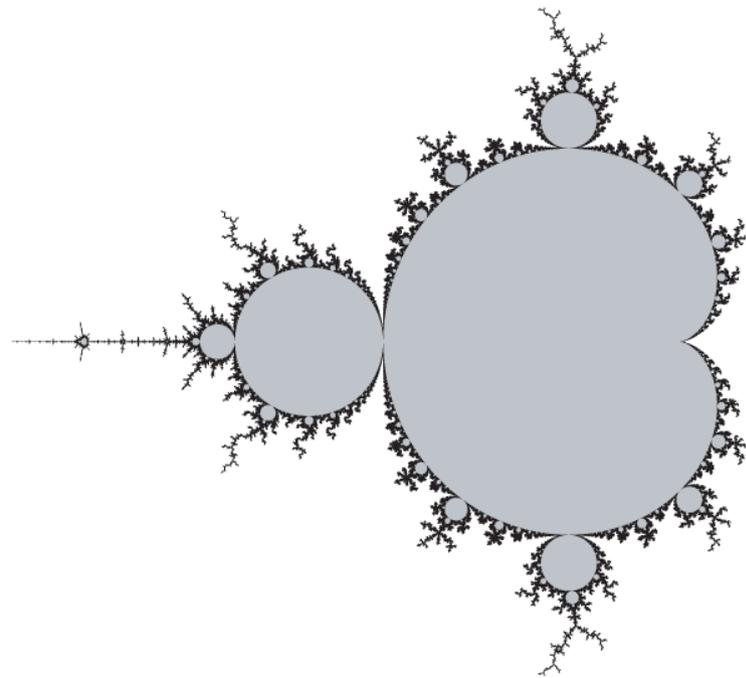


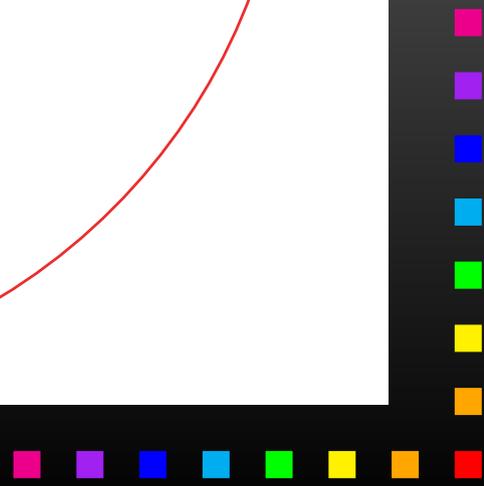
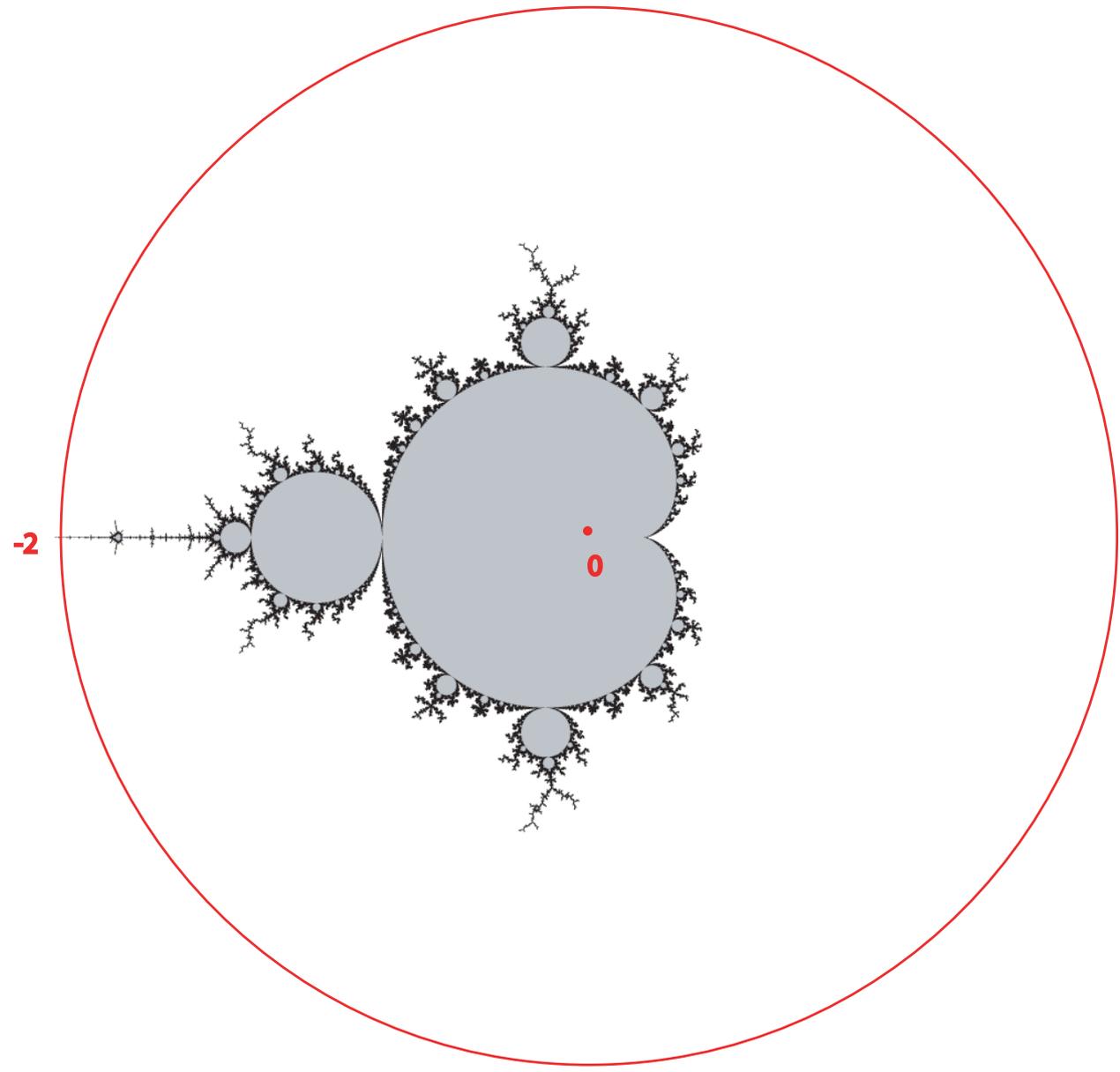
Double Mandelbrot, in the λ plane

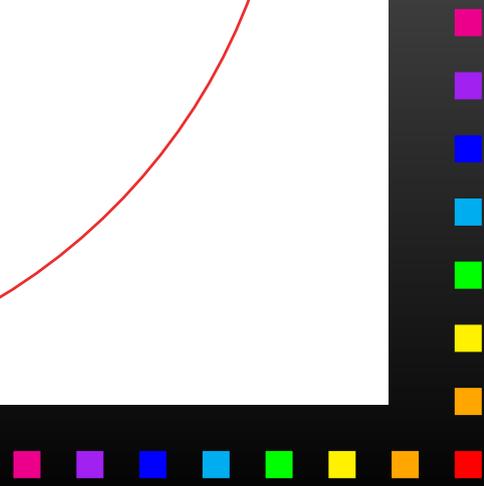
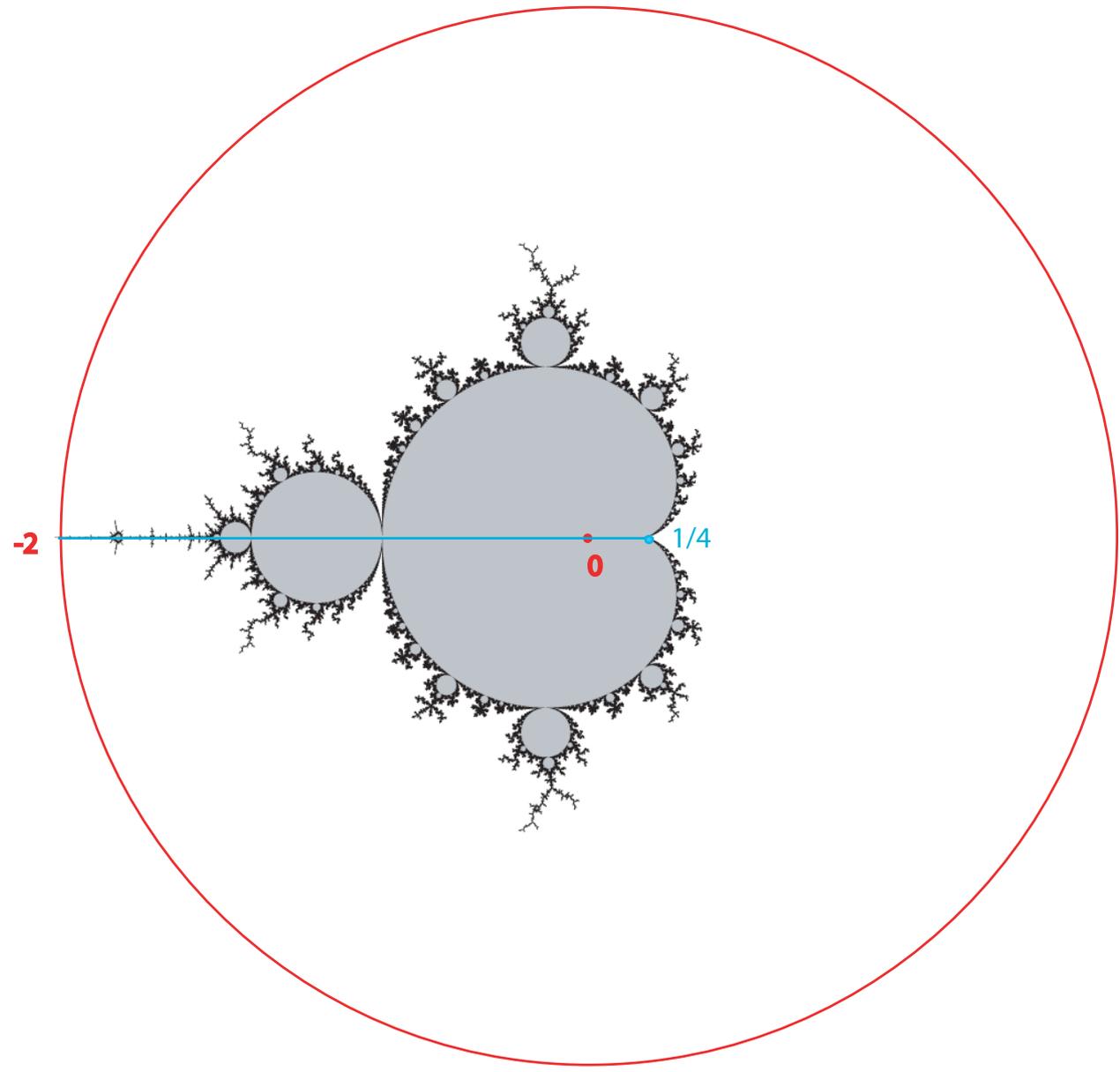


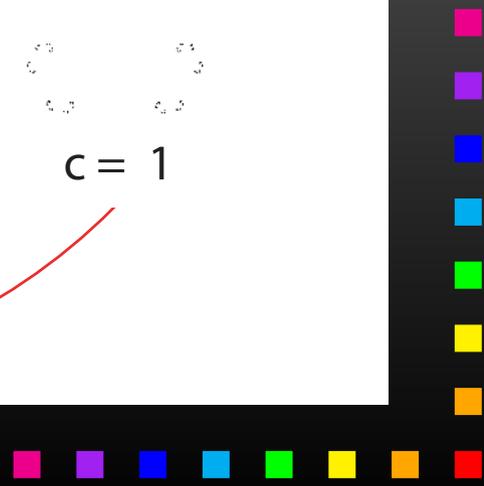
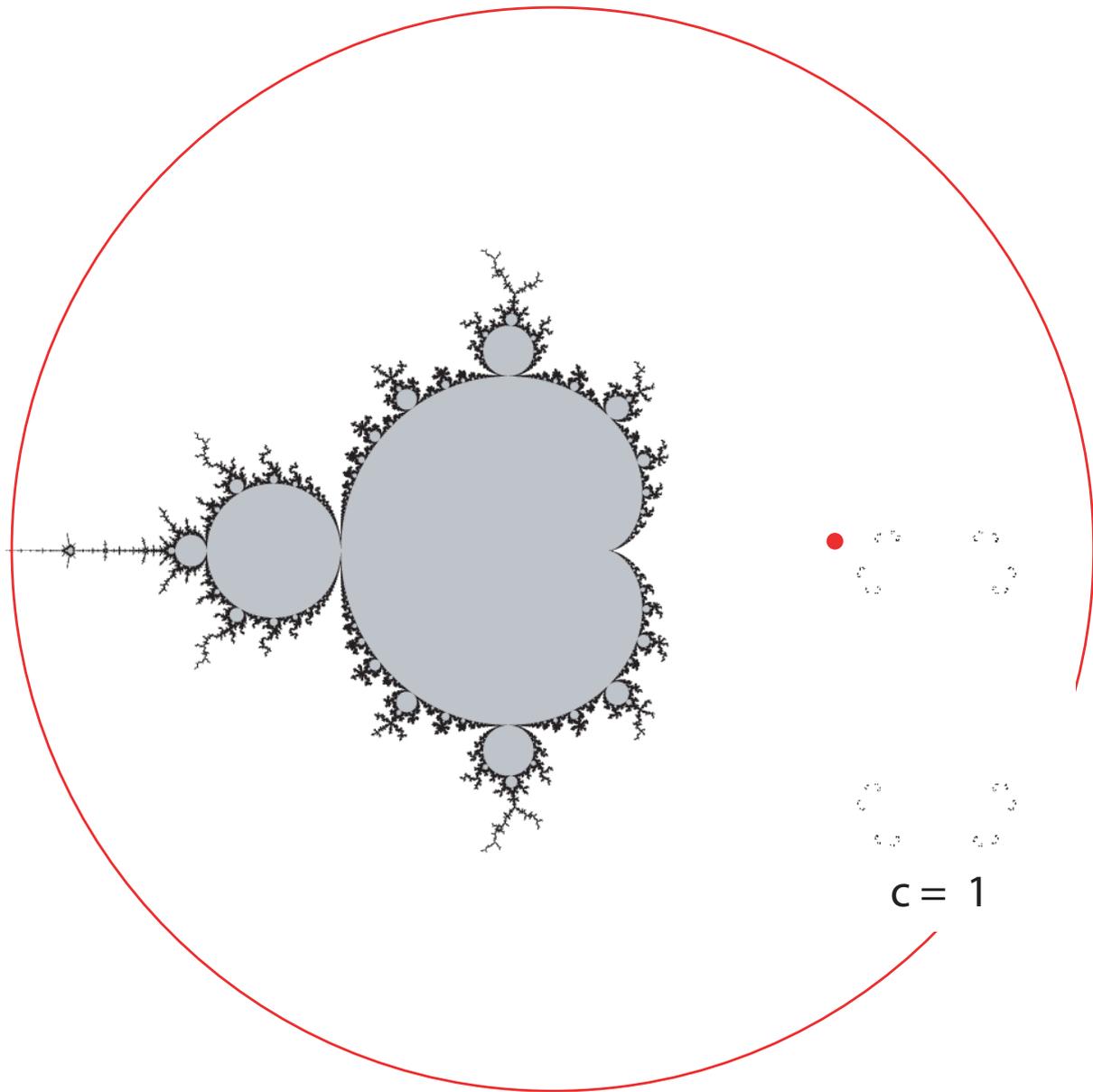
Double cover $\lambda \mapsto c$

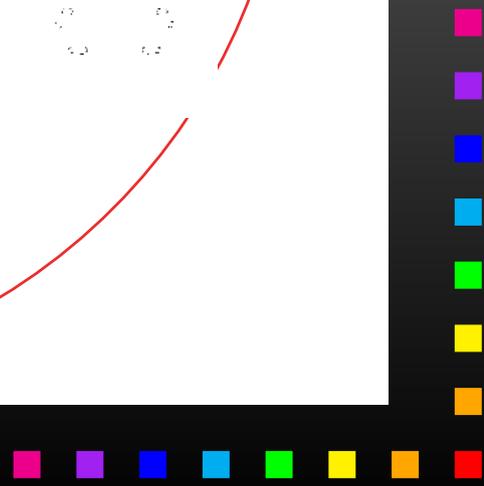
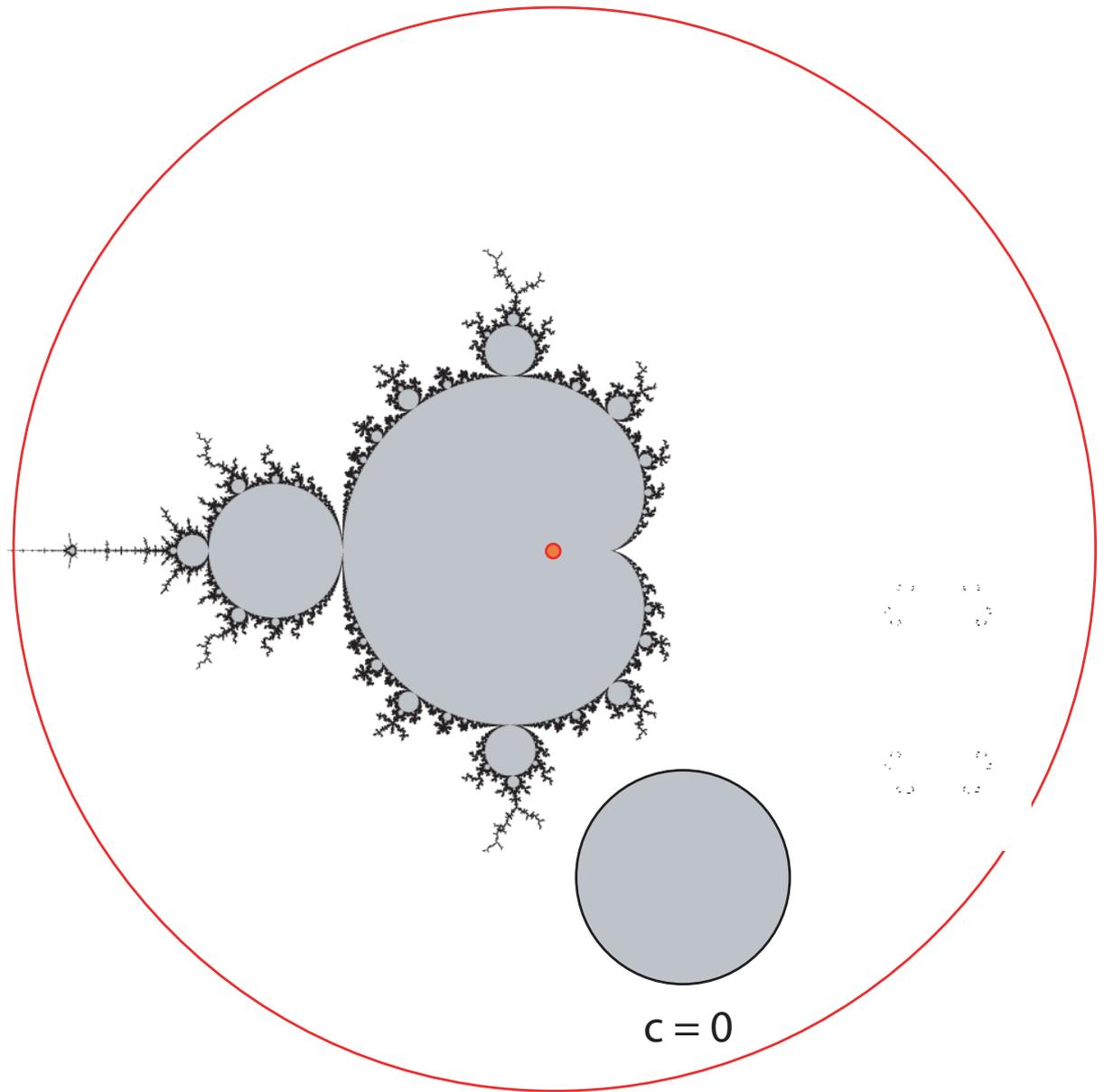


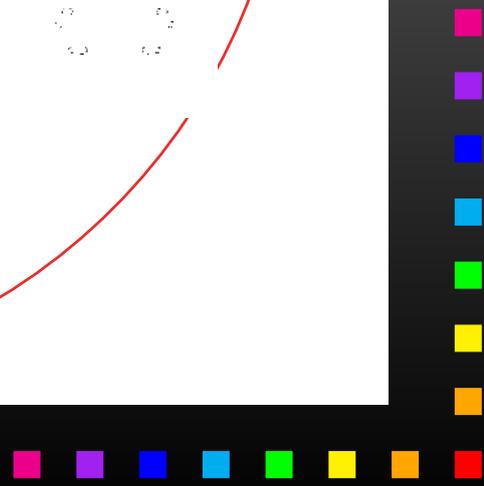
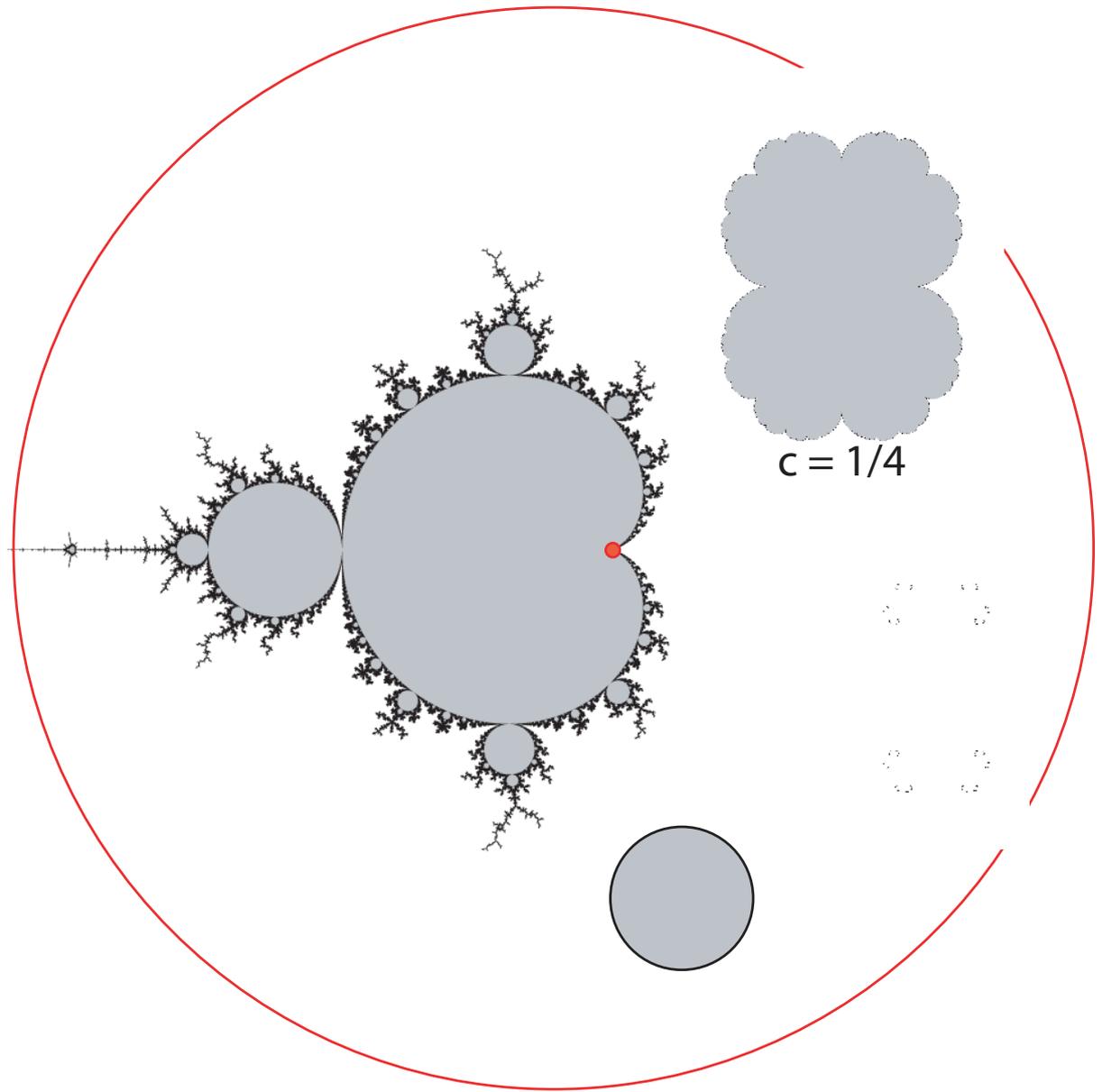


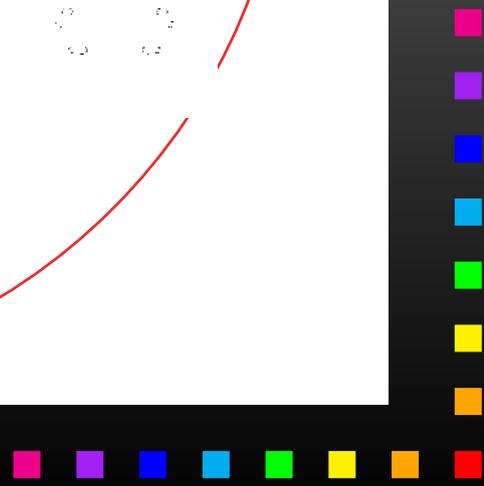
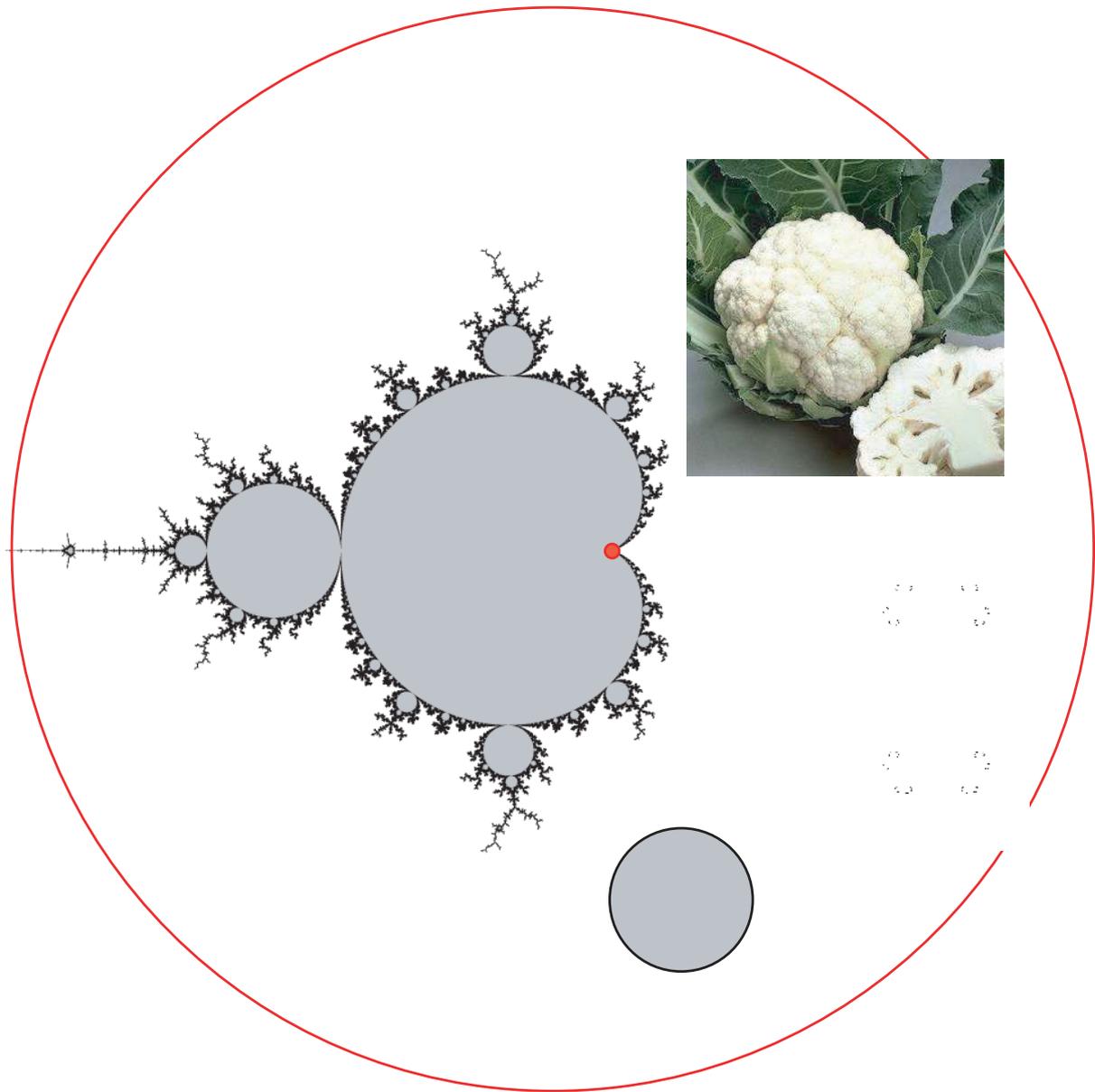


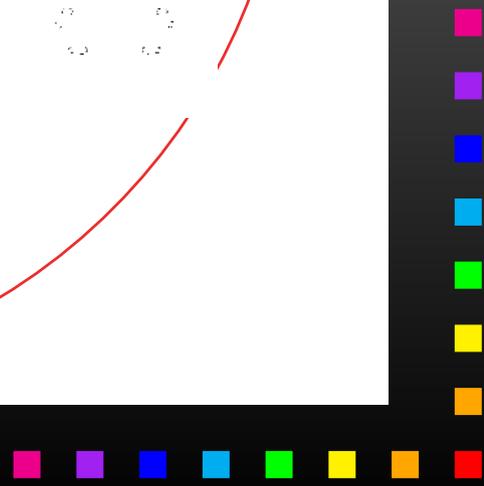
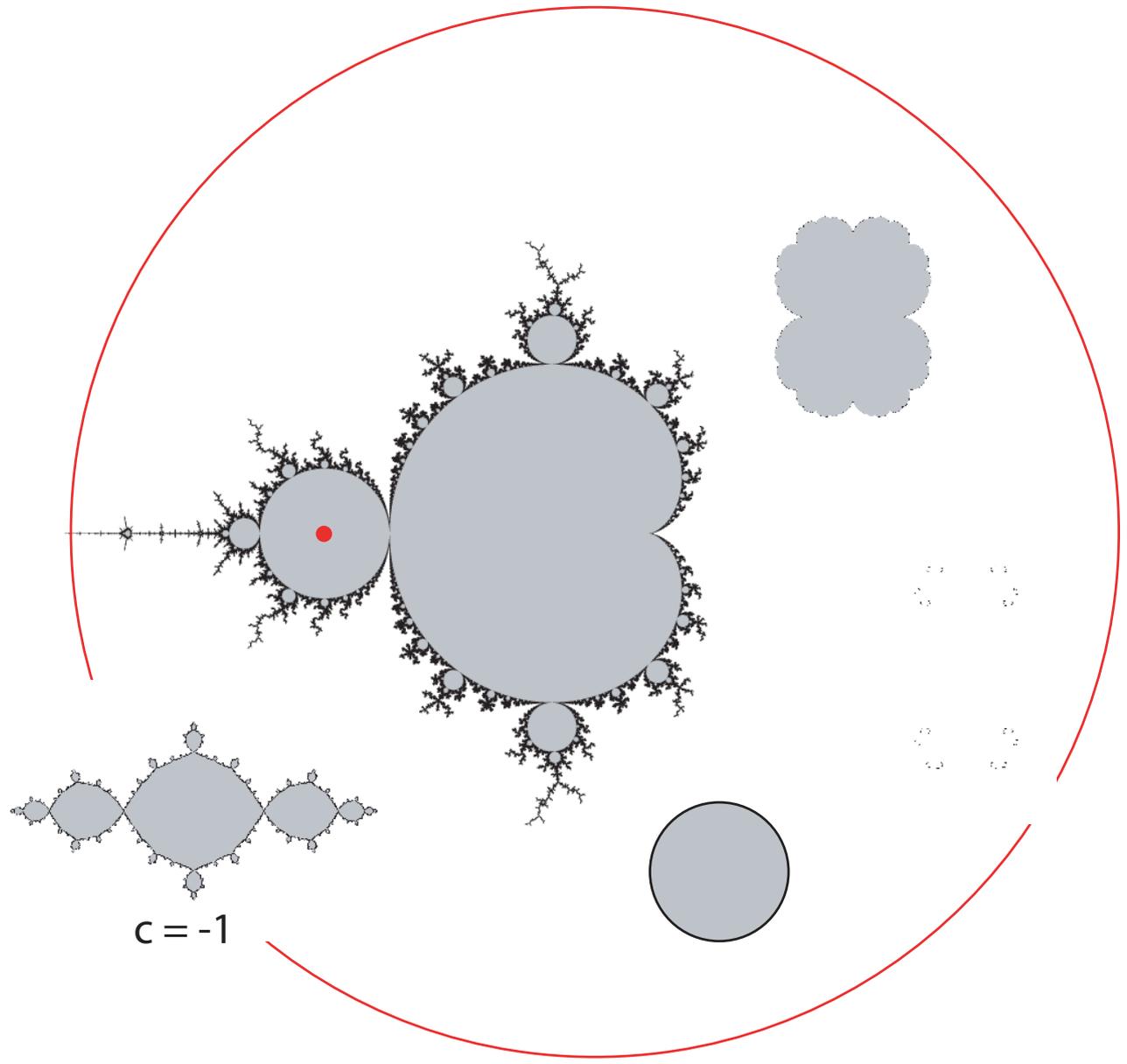


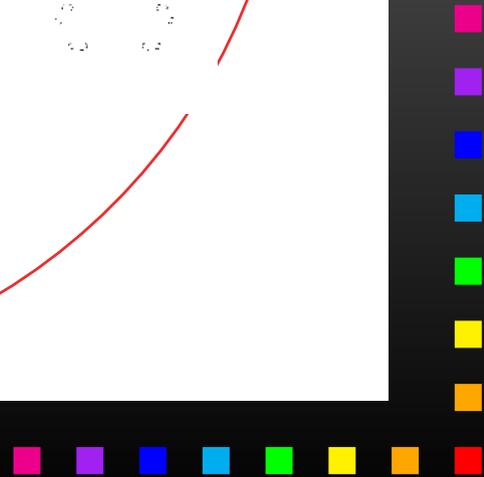
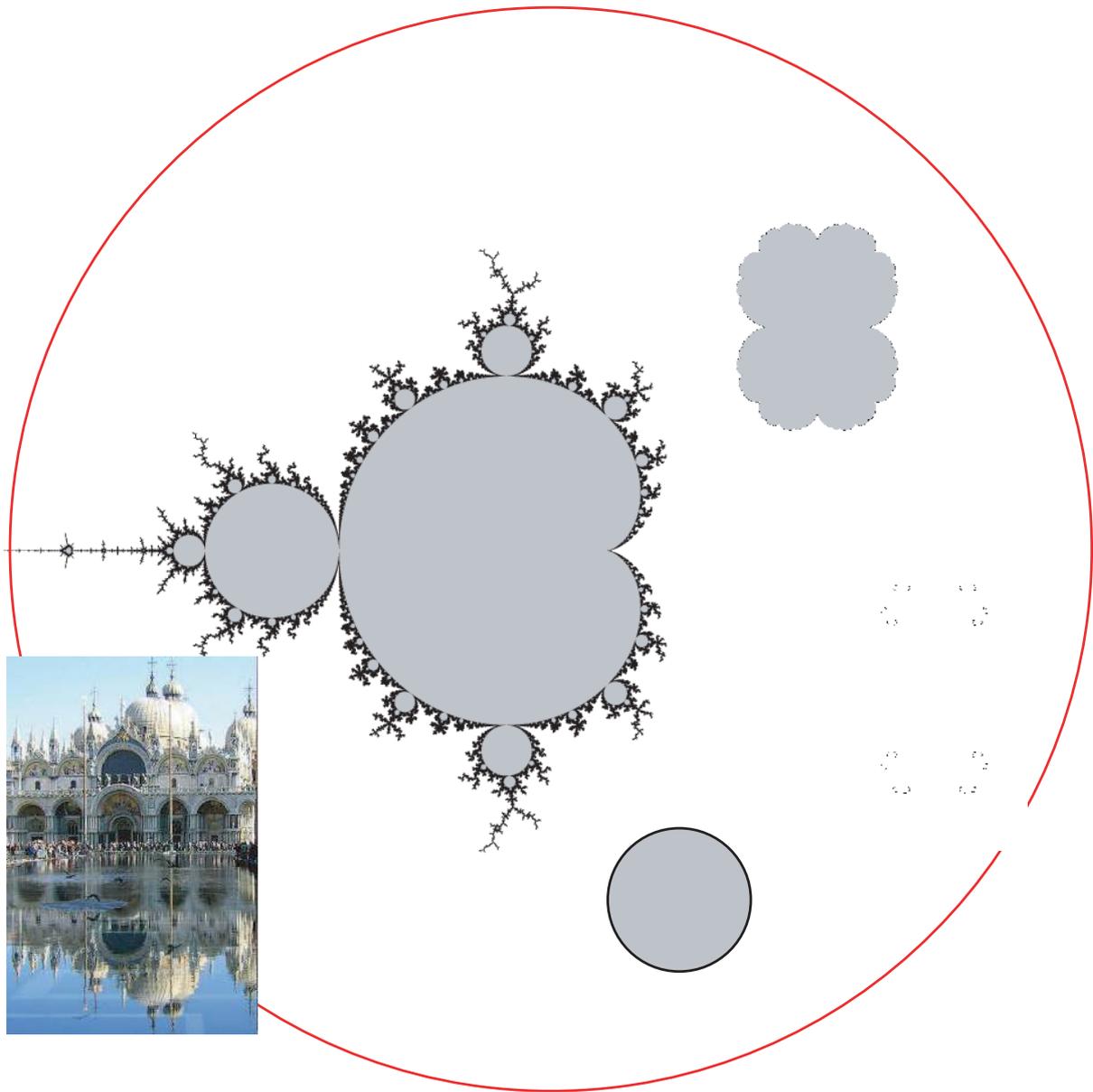


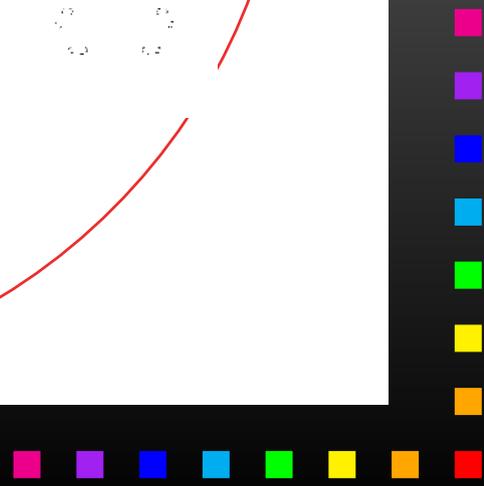
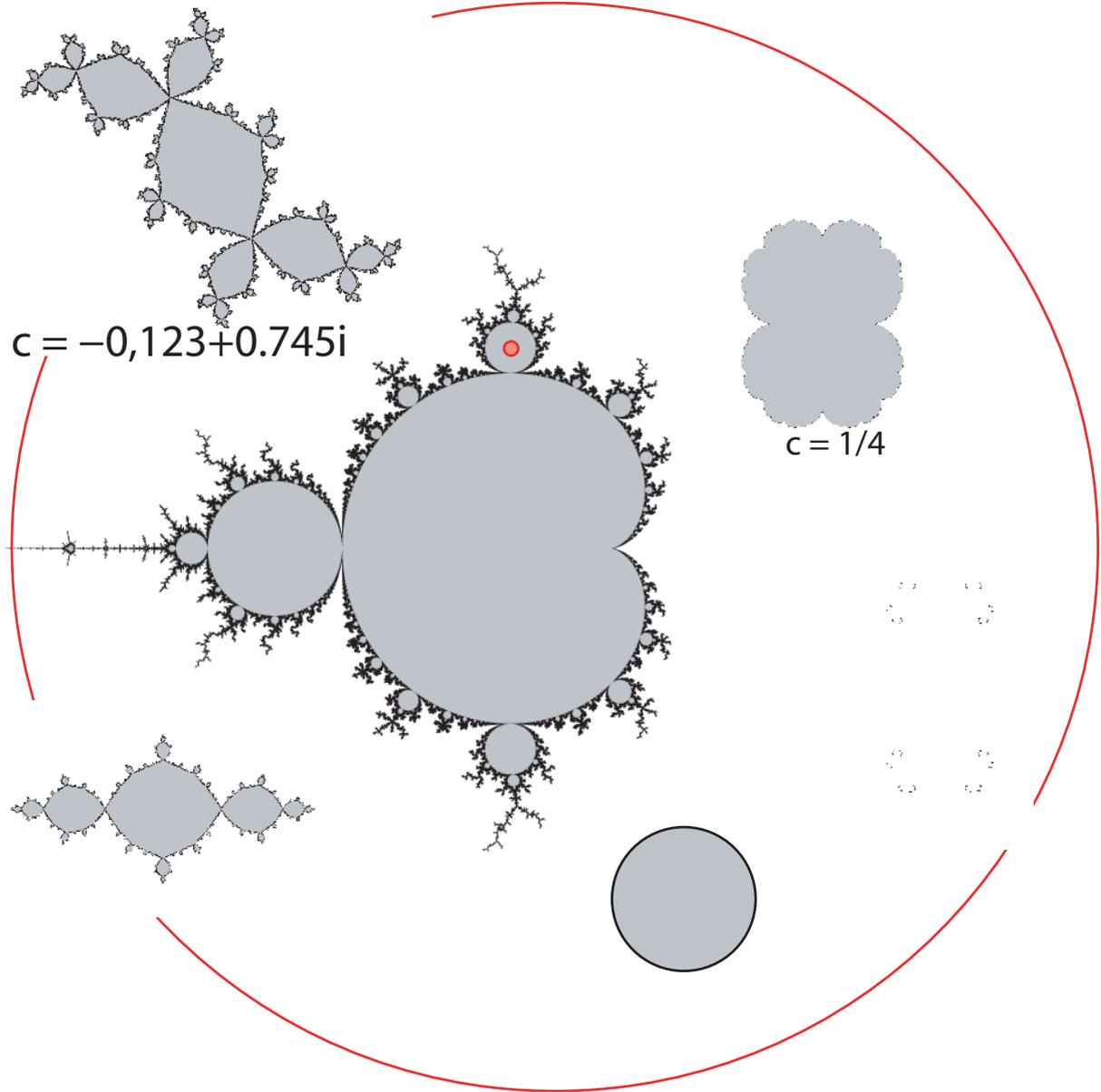


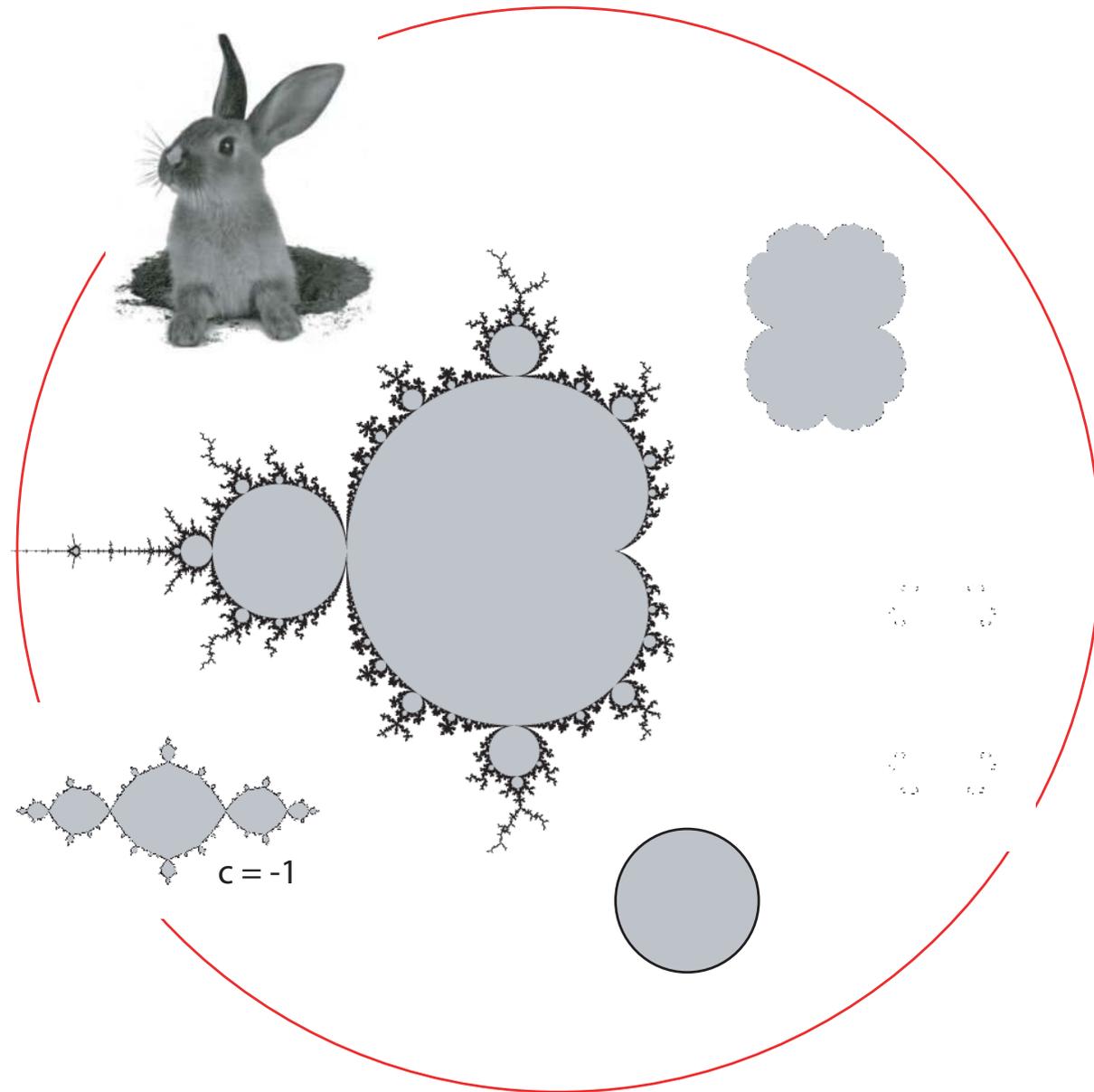












$c = -1$

